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THE ECONOMICS OF EXHAUSTIBLE RESOURCES

I. THE PECULIAR PROBLEMS OF MINERAL WEALTH

CONTEMPLATION of the world's disappearing supplies of minerals, forests, and other exhaustible assets has led to demands for regulation of their exploitation. The feeling that these products are now too cheap for the good of future generations, that they are being selfishly exploited at too rapid a rate, and that in consequence of their excessive cheapness they are being produced and consumed wastefully has given rise to the conservation movement. The method ordinarily proposed to stop the wholesale devastation of irreplaceable natural resources, or of natural resources replaceable only with difficulty and long delay, is to forbid production at certain times and in certain regions or to hamper production by insisting that obsolete and inefficient methods be continued. The prohibitions against oil and mineral development and cutting timber on certain government lands have this justification, as have also closed seasons for fish and game and statutes forbidding certain highly efficient means of catching fish. Taxation would be a more economic method than publicly ordained inefficiency in the case of purely commercial activities such as mining and fishing for profit, if not also for sport fishing. However, the opposition of those who are making the profits, with the apathy of everyone else, is usually sufficient to prevent the diversion into the public treasury of any con-

siderable part of the proceeds of the exploitation of natural resources.

In contrast to the conservationist belief that a too rapid exploitation of natural resources is taking place, we have the retarding influence of monopolies and combinations, whose growth in industries directly concerned with the exploitation of irreplaceable resources has been striking. If "combinations in restraint of trade" extort high prices from consumers and restrict production, can it be said that their products are too cheap and are being sold too rapidly?

It may seem that the exploitation of an exhaustible natural resource can never be too slow for the public good. For every proposed rate of production there will doubtless be some to point to the ultimate exhaustion which that rate will entail, and to urge more delay. But if it is agreed that the total supply is not to be reserved for our remote descendants and that there is an optimum rate of present production, then the tendency of monopoly and partial monopoly is to keep production below the optimum rate and to exact excessive prices from consumers. The conservation movement, in so far as it aims at absolute prohibitions rather than taxation or regulation in the interest of efficiency, may be accused of playing into the hands of those who are interested in maintaining high prices for the sake of their own pockets rather than of posterity. On the other hand, certain technical conditions most pronounced in the oil industry lead to great wastes of material and to expensive competitive drilling, losses which may be reduced by systems of control which involve delay in production. The government of the United States under the present administration has withdrawn oil lands from entry in order to conserve this asset, and has also taken steps toward prosecuting a group of California oil companies for conspiring to maintain unduly high prices, thus restricting production. Though these moves may at first sight appear contradictory in intent, they are really aimed at two distinct evils, a Scylla and Charybdis between which public policy must be steered.

In addition to these public questions, the economics of exhaustible assets presents a whole forest of intriguing problems. The

static-equilibrium type of economic theory which is now so well developed is plainly inadequate for an industry in which the indefinite maintenance of a steady rate of production is a physical impossibility, and which is therefore bound to decline. How much of the proceeds of a mine should be reckoned as income, and how much as return of capital? What is the value of a mine when its contents are supposedly fully known, and what is the effect of uncertainty of estimate? If a mine-owner produces too rapidly, he will depress the price, perhaps to zero. If he produces too slowly, his profits, though larger, may be postponed farther into the future than the rate of interest warrants. Where is his golden mean? And how does this most profitable rate of production vary as exhaustion approaches? Is it more profitable to complete the extraction within a finite time, to extend it indefinitely in such a way that the amount remaining in the mine approaches zero as a limit, or to exploit so slowly that mining operations will not only continue at a diminishing rate forever but leave an amount in the ground which does not approach zero? Suppose the mine is publicly owned. How should exploitation take place for the greatest general good, and how does a course having such an objective compare with that of the profit-seeking entrepreneur? What of the plight of laborers and of subsidiary industries when a mine is exhausted? How can the state, by regulation or taxation, induce the mine-owner to adopt a schedule of production more in harmony with the public good? What about import duties on coal and oil? And for these dynamical systems what becomes of the classic theories of monopoly, duopoly, and free competition?

Problems of exhaustible assets are peculiarly liable to become entangled with the infinite. Not only is there infinite time to consider, but also the possibility that for a necessity the price might increase without limit as the supply vanishes. If we are not to have property of infinite value, we must, in choosing empirical forms for cost and demand curves, take precautions to avoid assumptions, perfectly natural in static problems, which lead to such conditions.

While a complete study of the subject would include semi-replaceable assets such as forests and stocks of fish, ranging grad-

ually downward to such short-time operations as crop carry-overs, this paper will be confined in scope to absolutely irreplaceable assets. The forests of a continent occupied by a new population may, for purposes of a first approximation at least, be regarded as composed of two parts, of which one will be replaced after cutting and the other will be consumed without replacement. The first part obeys the laws of static theory; the second, those of the economics of exhaustible assets. Wild life which may replenish itself if not too rapidly exploited presents questions of a different type.

Problems of exhaustible assets cannot avoid the calculus of variations, including even the most recent researches in this branch of mathematics. However, elementary methods will be sufficient to bring out, in the next few pages, some of the principles of mine economics, with the help of various simplifying assumptions. These will later be generalized in considering a series of cases taking on gradually some of the complexities of the actual situation. We shall assume always that the owner of an exhaustible supply wishes to make the present value of all his future profits a maximum. The force of interest will be denoted by γ, so that $e^{-\gamma t}$ is the present value of a unit of profit to be obtained after time t, interest rates being assumed to remain unchanged in the meantime. The case of variable interest rates gives rise to fairly obvious modifications.[1]

2. FREE COMPETITION

Since it is a matter of indifference to the owner of a mine whether he receives for a unit of his product a price p_0 now or a price $p_0 e^{\gamma t}$ after time t, it is not unreasonable to expect that the price p will be a function of the time of the form $p = p_0 e^{\gamma t}$. This will not apply to monopoly, where the form of the demand function is bound to affect the rate of production, but is characteristic of completely free competition. The various units of the mineral are then to be thought of as being at any time all equally valuable, excepting for varying costs of placing them upon the market.

[1] As in "A General Mathematical Theory of Depreciation," by Harold Hotelling, *Journal of the American Statistical Association*, September, 1925.

They will be removed and used in order of accessibility, the most cheaply available first. If interest rates or degrees of impatience vary among the mine-owners, this fact will also affect the order of extraction. Here p is to be interpreted as the net price received after paying the cost of extraction and placing upon the market—a convention to which we shall adhere throughout.

The formula

$$p = p_0 e^{\gamma t} \tag{1}$$

fixes the relative prices at different times under free competition. The absolute level, or the value p_0 of the price when $t = 0$, will depend upon demand and upon the total supply of the substance. Denoting the latter by a, and putting

$$q = f(p,t)$$

for the quantity taken at time t if the price is p, we have the equation,

$$\int_0^T q\, dt = \int_0^T f(p_0 e^{\gamma t}, t)\, dt = a\,, \tag{2}$$

the upper limit T being the time of final exhaustion. Since q will then be zero, we shall have the equation

$$f(p_0 e^{\gamma t}, T) = 0 \tag{3}$$

to determine T.

The nature of these solutions will depend upon the function $f(p, t)$, which gives q. In accordance with the usual assumptions, we shall assume that it is a diminishing function of p, and depends upon the time, if at all, in so simple a fashion that the equations all have unique solutions.

Suppose, for example, that the demand function is given by

$$q = 5 - p\,, \qquad (0 \leq p \leq 5)$$
$$q = 0 \text{ for } p \geq 5\,,$$

independently of the time.

As q diminishes and approaches zero, p increases toward the value 5, which represents the highest price anyone will pay. Thus at time T,

$$p_0 e^{\gamma T} = 5 .$$

The relation (2) between the unknowns p_0 and T becomes in this case

$$a = \int_0^T (5 - p_0 e^{\gamma t}) dt = 5T - p_0 (e^{\gamma T} - 1)/\gamma .$$

Eliminating p_0, we have

$$a/5 = T + (e^{-\gamma T} - 1)/\gamma ,$$

that is,

$$e^{-\gamma T} = 1 + \gamma(a/5 - T) .$$

Now, if we plot as functions of T

$$y_1 = e^{-\gamma T}$$

and

$$y_2 = 1 + \gamma(a/5 - T) ,$$

we have a diminishing exponential curve whose slope where it crosses the y-axis is $-\gamma$, and a straight line with the same slope. The line crosses the y-axis at a higher point than the curve, since, when $T = 0$, $y_1 < y_2$. Hence there is one and only one positive value of T for which $y_1 = y_2$. This value of T gives the time of complete exhaustion. Clearly it is finite.

If the demand curve is fixed, the question whether the time until exhaustion will be finite or infinite turns upon whether a finite or infinite value of p will be required to make q vanish. For the demand function $q = e^{-bp}$, where b is a constant, the exploitation will continue forever, though of course at a gradually diminishing rate. If $q = a - \beta p$, all will be exhausted in a finite time. In general, the higher the price anticipated when the rate of production becomes extremely small, compared with the price for a more rapid production, the more protracted will be the period of operation.

3. MAXIMUM SOCIAL VALUE AND STATE INTERFERENCE

As in the static case, there is under free competition in the absence of complicating factors a certain tendency toward maximizing what might be called the "total utility" but is better called the "social value of the resource." For a unit of time this quantity may be defined as

$$u(q) = \int_0^q p(q)\, dq\,, \tag{4}$$

where the integrand is a diminishing function and the upper limit is the quantity actually placed upon the market and consumed. If future enjoyment be discounted with force of interest γ, the present value is

$$V = \int_0^T u[q(t)]e^{-\gamma t}\, dt\,.$$

Since $\int_0^T q\, dt$ is fixed, the production schedule $q(t)$ which makes V a maximum must be such that a unit increment in q will increase the integrand as much at one time as at another. That is,

$$\frac{d}{dq}\, u[q(t)]e^{-\gamma t}\,,$$

which by (4) equals $pe^{-\gamma t}$, is to be a constant. Calling this constant p_0, we have

$$p = p_0 e^{\gamma t}\,,$$

the result (1) obtained in considering free competition. That this gives a genuine maximum appears from the fact that the second derivative is essentially negative, owing to the downward slope of the demand curve.

This conclusion does not, of course, supply any more justification for laissez faire with the exploitation of natural resources than with other pursuits. It shows that the true basis of the conservation movement is not in any tendency inherent in competition under these ideal conditions. However, there are in extractive

industries discrepancies from our assumed conditions leading to particularly wasteful forms of exploitation which might well be regulated in the public interest. We have tacitly assumed all the conditions fully known. Great wastes arise from the suddenness and unexpectedness of mineral discoveries, leading to wild rushes, immensely wasteful socially, to get hold of valuable property.

Of this character is the drilling of "offset wells" along each side of a property line over a newly discovered oil pool. Each owner must drill and get the precious oil quickly, for otherwise his neighbors will get it all. Consequently great forests of tall derricks rise overnight at a cost of $50,000 or more each; whereas a much smaller number and a slower exploitation would be more economic. Incidentally, great volumes of natural gas and oil are lost because the suddenness of development makes adequate storage impossible.[a]

The unexpectedness of mineral discoveries provides another reason than wastefulness for governmental control and for special taxation. Great profits of a thoroughly adventitious character arise in connection with mineral discoveries, and it is not good public policy to allow such profits to remain in private hands. Of course the prospector may be said to have earned his reward by effort and risk; but can this be said of the landowner who discovers the value of his subsoil purely by observing the results of his neighbors' mining and drilling?

The market rate of interest γ must be used by an entrepreneur in his calculations, but should it be used in determinations of social value and optimum public policy? The use of $\int_0^q p\,dq$ as a measure of social value in a unit of time, whereas the smaller quantity pq would be the greatest possible profit to an owner for the same extraction of material, suggests that a similar integral be used in connection with the various rates of time-preference. There is, however, an important difference between the two cases in that the rate of interest is set by a great variety of forces, chiefly independent of the particular commodity and industry in question, and is not greatly affected by variations in the out-

[a] Cf. George W. Stocking, *The Oil Industry and the Competitive System*, Hart Schaffner and Marx prize essay (Houghton Mifflin, 1928).

put of the mine or oil well in question. It is likely, therefore, that in deciding questions of public policy relative to exhaustible resources, no large errors will be made by using the market rate of interest. Of course, changes in this rate are to be anticipated, especially in considering the remote future. If we look ahead to a distant time when all the resources of the earth will be near exhaustion, and the human race reduced to complete poverty, we may expect very high interest rates indeed. But the exhaustion of one or a few types of resources will not bring about this condition.

The discounting of future values of u may be challenged on the ground that future pleasures are ethically equivalent to present pleasure of the same intensity. The reply to this is that capital is productive, that future pleasures are uncertain in a degree increasing with their remoteness in time, and that V and u are concrete quantities, not symbols for pleasure. They measure the social value of the mine in the sense concerned with the total production of goods, but not properly its utility or the happiness to which it leads, since this depends upon the distribution of wealth, and is greater if the products of the mine benefit chiefly the poor than if they become articles of luxury. A platinum mine is of greater general utility when platinum is used for electrical and chemical purposes than when it is pre-empted by the jewelry trade. However, we must leave questions of distribution of wealth to be dealt with otherwise, perhaps by graded income and inheritance taxes, and consider the effects of various schedules of operation upon the total value of goods produced. It is for this reason that we are concerned with V.

The general question of how much of its income a people should save has been beautifully treated by F. P. Ramsey.[3]

Money metals, of course, occasion very special cause for public concern. Not only does gold production tend to unstabilize prices; but if the uses in the arts can be neglected, the costs of discovery, extraction, and transportation from the mine are, from the social standpoint, wasted.

Still a different reason for caution in deducing a laissez faire

[3] "A Mathematical Theory of Saving," *Economic Journal*, XXXVIII (1928), 543.

policy from the theoretical maximizing of V under "free" competition is that the actual conditions, even when competition exists, are likely to be far removed from the ideal state we have been postulating. A large producing company can very commonly affect the price by varying its rate of marketing. There is then something of the monopoly element, with a tendency toward undue retardation of production and elevation of price. This will be considered further in our last section. The monopoly problem

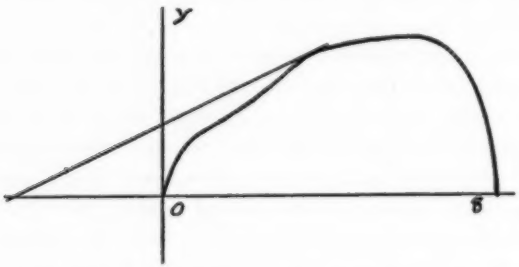

FIG. 1. $y=pq$. The tangent turns counterclockwise. The value of the mine is proportional to the distance to O from the intersection of the tangent with the q-axis.

of course extends also to non-extractive industries; but in dealing with exhaustible resources there are some features of special interest, which will now be examined.

4. MONOPOLY

The usual theory of monopoly prices deals with the maximum point of the curve

$$y=pq\,,$$

y being plotted as a function either of p or of q, each of these variables being a diminishing function of the other (Fig. 1). We now consider the problem of choosing q as a function of t, subject to the condition

$$\int_0^\infty q\,dt=a\,, \tag{5}$$

so as to maximize the present value,

$$J=\int_0^\infty qp(q)e^{-\gamma t}\,dt\,, \tag{6}$$

of the profits of the owner of a mine. We do not restrict q to be a continuous function of t, though p will be considered a continuous function of q with a continuous first derivative which is nowhere positive. The upper limit of the integrals may be taken as ∞ even if the exploitation is to take place only for a finite time T, for then $q=0$ when $t>T$.

This may or may not be considered a problem in the calculus of variations; some definitions of that subject would exclude our problem because no derivative is involved under the integral signs, though the methods of the science may be applied to it. However, the problem may be treated fairly simply by observing that

$$q\,p(q)e^{-\gamma t}-\lambda q\,, \tag{7}$$

where λ is a Lagrange multiplier, is to be a maximum for every value of t. We must therefore have

$$e^{-\gamma t}\frac{d}{dq}(pq)-\lambda=0\,, \tag{8}$$

and also

$$e^{-\gamma t}\frac{d^2}{dq^2}(pq)<0\,. \tag{9}$$

Evidently (8) may also be written

$$y'=\frac{d}{dq}(pq)=p+q\frac{dp}{dq}=\lambda e^{\gamma t}\,, \tag{10}$$

the contrast with the competitive conditions of the last section appearing in the term $q\,dp/dq$.

The constant λ is determined by solving (8) or (10) for q as a function of λ and t and substituting in (5). Upon integrating from 0 to T an equation will then be obtained for λ in terms of T and of

the amount a initially in the mine, which is here assumed to be known. The additional equation required to determine T is obtained by putting $q=0$ for $t=T$.

In general, if p takes on a finite value K as q approaches zero, $q\,dp/dq$ also remaining finite, (8) or (10) can be written

$$\frac{d(pq)}{dq}=Ke^{\gamma(t-T)},$$

Suppose, for example, that the demand function is

$$\begin{aligned} p&=(1-e^{-Kq})/q\,,\\ &=K-K^2q/2!+K^3q^2/3!-\dots\,, \end{aligned}$$

where K is a positive constant. For every positive value of q this expression is positive and has a negative derivative. As q approaches zero, p approaches K. We have

$$\begin{aligned} y&=pq=1-e^{-Kq}\,,\\ y'&=Ke^{-Kq}=\lambda e^{\gamma t} \end{aligned}$$

whence

$$q=(\log K/\lambda-\gamma t)/K\,,$$

this expression holding when t is less than T, the time of ultimate exhaustion. When $t=T$, q is of course zero. We have, therefore, putting $q=0$ for $t=T$,

$$\log K/\lambda=\gamma T;$$

and from (5)

$$\begin{aligned} a&=\int_0^T(\log K/\lambda-\gamma t)dt/K=\gamma\int_0^T(T-t)dt/K\\ &=\gamma T^2/2K \end{aligned}$$

so that

$$T=\sqrt{2Ka/\gamma}\,,$$

$$\log K/\lambda=\sqrt{2K\gamma a}\,,$$

giving finally,

$$q=\gamma(\sqrt{2Ka/\gamma}-t)/K\,.$$

5. GRAPHICAL STUDY: DISCONTINUOUS SOLUTIONS

The interpretation of (10) in terms of Figure 1 is that the rate of production is the abscissa of the point of tangency of a tangent line which rotates counterclockwise. The slope of this line is proportional to a sum increasing at compound interest.

Other graphical representations of the exhaustion of natural resources are possible. Drawing a curve giving $y'=d(pq)/dq$ as a function of q (Fig. 2), we have for the most profitable rate of

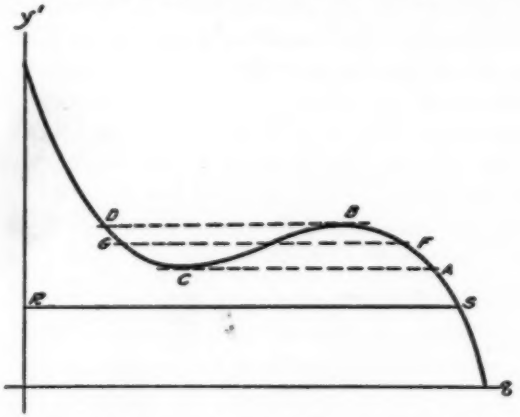

FIG. 2. *RS* rises with increasing speed. Its length is the rate of production and diminishes.

extraction the length of a horizontal line *RS* which rises like compound interest.

The waviness with which these curves have been drawn suggests that the solution obtained in this way is not unambiguous. Such waviness will arise if the demand function is, for example

$$p=b-(q-1)^3, \tag{11}$$

the derivative of which,

$$-3(q-1)^2,$$

is never positive. Here b is a constant, taken as 1 for Figure 2. For this demand function

$$y'=b-(4q-1)(q-1)^2.$$

When the rising line RS reaches the position AC, the point S whose abscissa represents the rate of production might apparently continue along the curve to B and then jump to D; or it might jump from A to C and then move on through D; or it might leave the arc AB at a point between A and B. At first sight there would seem to be another possibility, namely, to jump from A to C, to move up the curve to B, and then to leap to D. But this would mean increasing production for a period. This is never so profitable as to run through the same set of values of q in reverse order, for the total profit would be the same but would be received on the average more quickly if the most rapid production takes place at the beginning of the period. Hence we may regard q as always diminishing, though in this case with a discontinuity.

The values of q between which the leap is made in this case will be determined in § 10; it will be shown that the maximum profit will be reached if the monopolist moves horizontally from a certain point F on AB to a point G on CD.

6. VALUE OF A MINE MONOPOLY

To find the present value

$$J_{t_1}^{t_2} = \int_{t_1}^{t_2} pqe^{-\gamma t}dt$$

of the profits which are to be realized in any interval t_1 to t_2 during which the maximizing value of q is a continuous function of t, we integrate by parts:

$$J_{t_1}^{t_2} = -\frac{pqe^{-\gamma t}}{\gamma}\Big]_{t_1}^{t_2} + \frac{1}{\gamma}\int_{t_1}^{t_2} \frac{d(pq)}{dq}\frac{dq}{dt}\,e^{-\gamma t}\,dt\,.$$

When we put

$$y = pq \tag{12}$$

and apply (10), the last integral takes a simple form admitting direct integration. This gives, after applying (10) also to eliminate $e^{-\gamma t}$ from the first term,

$$J_{t_1}^{t_2} = \frac{\lambda}{\gamma}\left(q - \frac{y}{y'}\right)\Big]_{t_1}^{t_2}. \tag{13}$$

Now upon differentiating (12), we find

$$qy' = y + q^2 \frac{dp}{dq}.$$

Hence (13) can be written

$$\int_{t_1}^{t_2} = \frac{\lambda}{\gamma} \frac{q^2}{y'} \frac{dp}{dq} \Big]_{t_1}^{t_2}. \tag{14}$$

The expressions (13) and (14) provide very convenient means of computing the discounted profits. Their validity will be shown in § 10 to extend to cases in which q is discontinuous.

The expression

$$q - y/y'$$

which appears in (13) is, in terms of Figure 1, the difference between the abscissa and the subtangent of a point on the curve. It therefore equals the distance to the left of the origin of the point where a tangent to the curve meets the q-axis.

The value of the mine when $t=0$ is, in this notation, $\int_0^T$. It is λ/γ times the distance from the origin to the point of intersection with the negative x-axis of the initial tangent to the curve of monopoly profit.

7. RETARDATION OF PRODUCTION UNDER MONOPOLY

Although the rate of production may suffer discontinuities in spite of the demand function having a continuous derivative, these breaks will always occur during actual production, never at the end. Eventually q will trail off in a continuous fashion to zero. This means that the highest point of the curve of Figure 2 corresponds to $q=0$. To prove this, we use the monotonic decreasing character of p as a function of q, which shows that

$$y'(q) = p(q) + qp'(q)$$

is, for positive values of q, less than $p(q)$, and that this in turn is less than $p(0)$. Hence the curve rises higher at the y'-axis than for any level maximum at the right.

The duration of monopolistic exploitation is finite or infinite

according as y' takes on a finite or an infinite value when q approaches zero. This condition is a little different from that under competition, where a finite value of p as q approaches zero was found to be necessary and sufficient for a finite time. The two conditions agree unless p remains finite while $qp'(q)$ becomes infinite, in which case the demand curve reaches the p-axis and is tangent to it with contact of order higher than the first. In such a case the period of operation is finite under competition but infinite under monopoly. That this apparently exceptional case is quite likely to exist in fact is indicated by a study of the general properties of supply and demand functions applying the theory of frequency curves, a fascinating subject for which space will not be taken in this paper.

Such a study indicates that very high order contact of the demand curve with the p-axis is to be expected, and therefore that monopolistic exploitation of an exhaustible asset is likely to be protracted immensely longer than competition would bring about or a maximizing of social value would require. This is simply a part of the general tendency for production to be retarded under monopoly.

8. CUMULATED PRODUCTION AFFECTING PRICE

The net price p per unit of product received by the owner of a mine depends not only on the current rate of production but also on past production. The accumulated production affects both cost and demand. The cost of extraction increases as the mine goes deeper; and durable substances, such as gold and diamonds, by their accumulation influence the market. In considering this effect, the calculus of variations cannot be avoided; the following formulation in terms of this science will include as special cases the situations previously treated.

Let x be the amount which has been extracted from a mine, $q=dx/dt$ the current rate of production, and a the amount originally in the mine. Then p is a function of x as well as of q and t. The discounted profit at time $t=0$, which equals the value of the mine at that time, is

$$\int_0^\infty p(x, q, t)qe^{-\gamma t}\,dt\,. \tag{15}$$

If exhaustion is to come at a finite time T, we may suppose that $q=0$ for $t>T$, so that T becomes the upper limit. We put

$$f(x, q, t)=pqe^{-rt}.$$

Then the owner of the mine (who is now assumed to have a monopoly) cannot do better than to adjust his production so that

$$\frac{\partial f}{\partial x}-\frac{d}{dt}\frac{\partial f}{\partial q}=0.$$

In case f does not involve x, the first term is zero and the former case of monopoly is obtained.

In general the differential equation is of the second order in x, since $q=dx/dt$, and so requires two terminal conditions. One of these is $x=0$ for $t=0$. The other end of the curve giving x as a function of t may be anywhere on the line $x=a$, or the curve may have this line as an asymptote. This indefiniteness will be settled by invoking again the condition that the discounted profit is a maximum. The "transversality condition" thus obtained,

$$f-q\frac{\partial f}{\partial q}=0,$$

that is,

$$q^2\frac{\partial p}{\partial q}=0,$$

is equivalent to the proposition that, if p always diminishes when q increases, the curve is tangent or asymptotic to the line $x=a$. Thus ultimately q descends continuously to zero.

Suppose, for example, that q, x, and t all affect the net price *linearly*. Thus

$$p=a-\beta q-cx+gt.$$

Ordinarily a, β, and c will be positive, but g may have either sign. The growth of population and the rising prices to consumers of competing exhaustible goods would lead to a positive value of g. On the other hand, the progress of science might lead to the grad-

ual introduction of new substitutes for the commodity in question, tending to make g negative. The exhaustion of complementary commodities would also tend toward a negative value of g.

The differential equation reduces, for this linear demand function, to the linear form

$$2\beta\frac{d^2x}{dt^2}-2\beta\gamma\frac{dx}{dt}-c\gamma x=-g\gamma t+g-a\gamma\,.$$

Since β, c, and γ are positive, the roots of the auxiliary equation are real and of opposite signs. Let m denote the positive and $-n$ the negative root. Since

$$m-n=\gamma\,,$$

m is numerically greater than n. The solution is

$$x=Ae^{mt}+Be^{-nt}+gt/c-2\beta g/c^2-g/c\gamma+a/c\,,$$

whence

$$q=Ame^{mt}-Bne^{-nt}+g/c\,.$$

Since $x=0$ when $t=0$,

$$A+B-2\beta g/c^2-g/c\gamma+a/c=0\,.$$

Since $x=a$ and $q=0$ at the time T of ultimate exhaustion,

$$\begin{aligned}Ae^{mT}+Be^{-nT}+gT/c-2\beta g/c^2-g/c\gamma+a/c-a&=0\,,\\ Ame^{mT}-Bne^{-nT}+g/c&=0\,.\end{aligned}$$

From these equations A and B are eliminated by equating to zero the determinant of their coefficients and of the terms not containing A or B. After multiplying the first column by e^{-mT} and the second by e^{nT}, this gives:

$$\Delta=\begin{vmatrix} e^{-mT} & e^{nT} & -2\beta g/c^2-g/c\gamma+a/c \\ 1 & 1 & gT/c-2\beta g/c^2-g/c\gamma+a/c-a \\ m & -n & g/c \end{vmatrix}=0\,.$$

Expanding and using the relations $m-n=\gamma$, and $mn=c\gamma/2\beta$, we have for Δ and its derivative with respect to T,

$$\Delta=(e^{-mT}-e^{nT})g/c+(ne^{-mT}+me^{nT})(gT/c-2\beta g/c^2-g/c\gamma+a/c-a)$$
$$+(m+n)(2\beta g/c^2+g/c\gamma-a/c)\,,$$

$$\Delta'=(e^{nT}-e^{-mT})[T-1/\gamma+(a-ac)/g]g\gamma/2\beta\,,$$

the last expression being useful in applying Newton's method to find T. Obviously, the derivative changes sign for only one value of T; for this value Δ has a minimum if g is positive, a maximum if g is negative.

We may measure time in such units that γ, the force of interest, is unity. If money is worth 4 per cent, compounded quarterly, the unit of time will then be about 25 years and 1 month. With this convention let us consider an example in which there is an upward secular trend in the price consumers are willing to pay: take $a=100$, $\beta=1$, $c=4$, $g=16$, and $a=10$. The net amount received per unit is in this case

$$p=100-q-4x+16t\,.$$

Substituting the values of the constants, and noting that $m=2$ and $n=1$, we have

$$\Delta=\begin{vmatrix} e^{-2T} & e^{T} & 19 \\ 1 & 1 & 4T+9 \\ 2 & -1 & 4 \end{vmatrix}=(8T+14)e^{T}+(4T+13)e^{-2T}-57\,,$$

$$\Delta'=(e^{T}-e^{-2T})(8T+22)\,.$$

Evidently $\Delta<0$ for $T=0$, $\Delta=+\infty$ for $T=\infty$, and $\Delta'>0$ for all positive values of T. Hence $\Delta=0$ has one and only one positive root.

For the trial value $T=1$ we have

$$\Delta=5.10\,,\qquad \Delta'=77.5\,.$$

Applying to T the correction $-\Delta/\Delta' = -.07$ roughly, we take $T = .93$ as a second approximation. For this value of T,

$$\Delta = -.06\,, \qquad \Delta' = 70.0\,,$$

whence $-\Delta/\Delta' = .001$.

The most profitable schedule of extraction will therefore exhaust the mine in about 0.931 unit of time, or about 23 years and 4 months, perhaps a surprisingly short time in view of the prospect of obtaining an indefinitely higher price in the future, at the rate of increase of 16 per unit of time.

In order that the time of working a mine be infinite, it is necessary not only that the price shall increase indefinitely but that it shall ultimately increase at least as fast as compound interest.

The last two equations for determining A and B now become, since $e^{2T} = 6.4366$ and $e^{-T} = .3942$,

$$\begin{aligned} 6.4366A + .3942B + 12.724 &= 0 \\ 12.8732A - .3942B + 4 &= 0\,. \end{aligned}$$

Hence $A = -.866$, $B = -18.13$; so that

$$x = -.866e^{2t} - 18.13e^{-t} + 4t + 19\,.$$

As a check we observe that this expression for x vanishes when $t = 0$.

Differentiating, we have

$$q = -1.732e^{2t} + 18.13e^{-t} + 4\,,$$

showing how the rate of production begins at 20.40 and gradually declines to zero. Substitution in the assumed expression for the net price gives

$$\begin{aligned} p &= 100 - q - 4x + 16t \\ &= 20 + 5.196e^{2t} + 54.39e^{-t}\,, \end{aligned}$$

showing a decline from 79.60 at the beginning to 74.90 at exhaustion, owing to the greater cost of extracting the deeper parts of

the deposit. The buyer of course pays an increasing, not a decreasing price, namely,

$$p+4x=100-q+16t$$
$$=96+1.732e^{2t}-18.13e^{-t}+16t.$$

This increases from 79.60 to 114.90.

9. THE OPTIMUM COURSE

To examine the course of exploitation of a mine which would be best socially, in contrast with the schedule which a well-informed but entirely selfish owner would adopt, we generalize the considerations of § 3. Instead of the rate of profit pq, we must now deal with the social return per unit of time,

$$u=\int_0^q p(x, q, t)dq\,,$$

x and t being held constant in the integration. Taking again the market rate of interest as the appropriate discount factor for future enjoyments, we set

$$F=ue^{-\gamma t}\,,$$

and inquire what curve of exploitation will make the total discounted social value,

$$V=\int F dt$$

a maximum.

The characteristic equation

$$\frac{\partial F}{\partial x}-\frac{d}{dt}\frac{\partial F}{\partial q}=0$$

reduces to

$$\frac{\partial p}{\partial q}\frac{d^2x}{dt^2}+\frac{\partial p}{\partial x}\frac{dx}{dt}-\gamma p=\frac{\partial u}{\partial x}-\frac{\partial p}{\partial t}\,.$$

The initial condition is $x=0$ for $t=0$. The other end-point of the curve is movable on the line $x-a=0$, a being the amount originally in the mine. The transversality condition,

$$F-q\frac{\partial F}{\partial q}=0\,,$$

reduces to

$$u-pq=0\,.$$

This is satisfied only for $q=0$, for otherwise we should have the equation

$$p=\frac{1}{q}\int_0^q p\,dq\,,$$

stating that the ultimate price is the mean of the potential prices corresponding to lower values of q. Since p is assumed to decrease when q increases, this is impossible. Even if $\partial p/\partial q$ is zero in isolated points, the equation will be impossible if, as is always held, this derivative is elsewhere negative. Hence $q=0$ at the time of exhaustion.

If, as in § 8, we suppose the demand function linear,

$$p=\alpha-\beta q-cx+gt\,,$$

the characteristic equation becomes

$$\beta\frac{d^2x}{dt^2}-\beta\gamma\frac{dx}{dt}-c\gamma x=-g\gamma t+g-\alpha\gamma\,.$$

This differs from the corresponding equation for monopoly only in that β is here replaced by $\beta/2$. In a sense, this means that the decline of price, or marginal utility, with increase of supply counts just twice as much in affecting the rate of production, when this is in the control of a monopolist, as the public welfare would warrant.

The analysis of § 8 may be applied to this case without any qualitative change. The values of m and n depend on β, and are therefore changed. The time T until ultimate exhaustion will be

reduced, if social value rather than monopoly profit is to be maximized. For the numerical example given, T was found to be 0.931 unit of time under monopoly. Repeating the calculation for the case in which maximum social value is the goal, we find as the best value only 0.6741 unit of time.

For different values of the constants, even with a linear demand function, the mathematics may be less simple. For example, the equation $\Delta = 0$ may have two positive roots instead of one. This will be the case if the numerical illustration chosen be varied by supposing that the sign of g is reversed, owing to the progressive discovery of substitutes, the direct effect of passage of time being then to decrease instead of increase the price. In such cases a further examination is necessary of the two possible curves of development, to determine which will yield a greater monopoly profit or total discounted social value, according to our object.

10. DISCONTINUOUS SOLUTIONS

Even if the rate of production q has a discontinuity, as in the example of § 5, the condition that $\int f\,dt$ shall be a maximum requires that each of the quantities

$$\frac{\partial f}{\partial q}, \qquad f - q\frac{\partial f}{\partial q},$$

must nevertheless be continuous.[4] This will be true whether f stands for discounted monopoly profit or discounted total utility.

The equation (8) on p. 147, may be written

$$\frac{\partial f}{\partial q} = \lambda,$$

which shows, since the left-hand member is continuous, that λ must have the same value before and after the discontinuity.

When p is a function of q alone, the two continuous quantities may be written in the notation of § 4, $y'e^{-\gamma t}$ and $(y - qy')\,e^{-\gamma t}$,

[4] C. Caratheodory, "Über die diskontinuirlichen Lösungen in der Variationsrechnung," thesis, Göttingen, 1904, p. 11. The condition that the first of these quantities must be continuous is given in the textbooks, but for some reason the second is generally omitted.

which shows that y' and $y-qy'$ are continuous. Thus the expression $\lambda\,(q-y/y')$ appearing in (13), p. 150, is continuous. Consequently the expressions (13) or (14) pertaining to the different time-intervals may simply be added to obtain an expression of the same form. Hence the present value of the discounted future profits of the mine—and therefore of the mine—is in such cases the difference between the values of

$$\lambda(q-y/y')/\gamma$$

at present and at the time of exhaustion.

We are now ready to answer such questions as that raised at the end of § 5 as to the location of the discontinuity there shown to exist in the most profitable schedule of production when the demand function is

$$p=b-(q-1)^3\,.$$

Since in this case

$$f=pqe^{-\gamma t}=[bq-q(q-1)^3]e^{-\gamma t}\,,$$

the two quantities

$$b-(4q-1)(q-1)^3\,,$$
$$3q^2(q-1)^3\,,$$

are continuous. Consequently

$$(4q-1)(q-1)^3$$

and

$$q^2(q-1)^3$$

are continuous. If q_1 denote the rate of production just before the sudden jump and q_2 the initial rate after it, this means that

$$(4q_1-1)(q_1-1)^3=(4q_2-1)(q_2-1)^3$$
$$q_1^2(q_1-1)^3=q_2^2(q_2-1)^3\,.$$

The only admissible solution is:

$$q_1=(3+\sqrt{3})/4=1.1830\,,\qquad q_2=(3-\sqrt{3})/4=0.31699\,.$$

11. TESTS FOR A TRUE MAXIMUM

The equations which have been given for finding the production schedule of maximum profit or social value are necessary, not sufficient, conditions for maxima, like the vanishing of the first derivative in the differential calculus. We must also consider more definitive tests.

The integrals which have arisen in the problems of exhaustible assets are to be maxima, not necessarily for the most general type of variation conceivable for a curve, but only for the so-called "special weak" variations. The nature of the economic situation seems to preclude all variations which involve turning time backward, increasing the rate of production, maintaining two different rates of production at the same time, or varying production with infinite rapidity. Extremely sudden increases in production usually involve special costs which will be borne only under unexpected conditions, and are to be avoided in long-term planning. Likewise sudden decreases involve social losses of great magnitude such as unemployment, which even a selfish monopolist will often try to prevent. This will be considered further in the next section. It is indeed possible that in some special cases these "strong" variations might take on some economic significance, but such a situation would involve forces of a different sort from those with which economic theory is ordinarily concerned.

The critical tests which must be applied are by the foregoing considerations reduced to two—those of Legendre and Jacobi.[5] The Legendre test requires, in order that the total discounted utility or social value (§ 9) shall be a maximum, that

$$\frac{\partial^2 u}{\partial q^2}=\frac{\partial p}{\partial q}<0\,,$$

a condition which is always held to obtain save in exceptional cases. In order that the chosen curve shall yield a genuine maximum for a monopolist's profit, the Legendre test requires that

$$\frac{\partial^2(pq)}{\partial q^2}=2\,\frac{\partial p}{\partial q}+q\,\frac{\partial^2 p}{\partial q^2}<0\,.$$

[5] A. R. Forsyth, *Calculus of Variations* (Cambridge, 1927), pp. 17–28.

This means that the curve of Figure 1 is convex upward at all points touched by the turning tangent. The re-entrant portions, if any, are passed over, producing discontinuities in the rate of production.

When the solution of the characteristic equation has been found in the form

$$x=\varphi(t, A, B),$$

A and B being arbitrary constants, the Jacobi test requires that

$$\frac{\partial\varphi/\partial A}{\partial\varphi/\partial B}$$

shall not take the same value for two different values of t. For the example of § 8 this critical quantity is simply $e^{(m+n)t}$, which obviously satisfies the test. The solution represents a real, not an illusory maximum for the monopolist's profit. The like is true for the schedule of production maximizing the total discounted utility with the same demand function. Each case must, however, be examined separately, as the test might show in some instances that a seeming maximum could be improved.

12. THE NEED FOR STEADINESS IN PRODUCTION

The demand function giving p may involve not only the rate of production q, but also the rate of change q' of q. Such a condition would display a duality with that considered by C. F. Roos[6] and G. C. Evans,[7] who hold that the quantity of a commodity which can be sold per unit of time depends ordinarily upon the rate of change of the price, as well as upon the price itself. If p is a function of x, q, q', and t, the maximum of monopoly profit or of social value can only be obtained if the course of exploitation satisfies a fourth-order differential equation.

[6] "A Dynamical Theory of Economics," *Journal of Political Economy*, XXXV (1927), 632, and references there given; also "A Mathematical Theory of Depreciation and Replacement," *American Journal of Mathematics*, L (1928), 147.

[7] "The Dynamics of Monopoly," *American Mathematical Monthly*, Vol. XXXI (1924); also *Mathematical Introduction to Economics* (McGraw-Hill Book Co., 1930).

More generally we might suppose that p and its rate of change p' are connected with x, q, q', and t by a relation

$$\varphi(p, p', x, q, q', t)=0.$$

This presents a Lagrange problem, which can be dealt with by known methods.[8] A further generalization is to suppose that the price, the quantity, and their derivatives are subject to a relation in the nature of a demand function which also involves an integral or integrals giving the effect of past prices and rates of consumption.[9]

Capital investment in developing the mine and industries essential to it is a source of a need for steady production; the desirability of regular employment for labor is another. Under the term "capital" might possibly be included the costs, both to employers and to laborers, in drawing laborers to the mine from other places and occupations. The returning of these laborers to other occupations as production declines would have to be reckoned as part of the social cost. Whether this would enter into the mine-owner's costs would probably depend upon whether the laborers have at the beginning sufficient information and bargaining power to insist upon compensation for the cost to them of the return shift.

Problems in which the fixity of capital investment plays a part in determining production schedules may be dealt with by introducing new variables $x_1, x_2, \ldots$, to represent the various types of capital investment involved. In so far as these variables are continuous, the problem is that of maximizing an integral involving x, x_1, x_2, ..., and their derivatives, using well-known methods. The simultaneous equations

$$\frac{\partial f}{\partial x_i}-\frac{d}{dt}\frac{\partial f}{dx_i}=0 \qquad (i=0, 1, 2, \ldots; x_0=x; x_i'=dx_i/dt)$$

[8] Cf. G. A. Bliss, "The Problem of Lagrange in the Calculus of Variations" (mimeographed by O. E. Brown), (University of Chicago Bookstore).

[9] See "Generalized Lagrange Problems in the Calculus of Variations," by C. F. Roos, *Transactions of the American Mathematical Society*, XXX, (1927), 360.

are necessary for a maximum. The depreciation of mining equipment raises considerations of this kind.

The cases considered in the earlier part of this paper all led to solutions in which the rate of production of a mine always decreases. By considering the influence of fixed investments and the cost of accelerating production at the beginning, we may be led to production curves which rise continuously from zero to a maximum, and then fall more slowly as exhaustion approaches. Certain production curves of this type have been found statistically to exist for whole industries of the extractive type, such as petroleum production.[10]

13. CAPITAL VALUE TAXES AND SEVERANCE TAXES

An unanticipated tax upon the value of a mine will have no effect other than to transfer to the government treasury a part of the mine-owner's income. An anticipated tax at the rate α per year and payable continuously will have the same effects upon the value of the mine and the schedule of production as an increase of the force of interest by α. This we shall now prove.

From the income pq from the mine at time t must now be deducted the tax, $\alpha J(t)$. Consequently the value at time τ is

$$J(\tau)=\int_{\tau}^{T}[pq-\alpha J(t)]e^{-\gamma(t-\tau)}dt\,.$$

This integral equation in J reduces by differentiation to a differential equation:

$$J'(\tau)=-pq+\alpha J(\tau)+\gamma J(\tau)\,.$$

The solution is found by well-known methods. The constant of integration is evaluated by means of the condition that $J(T)=0$. We have:

$$J(\tau)=\int_{\tau}^{T}pqe^{-(\alpha+\gamma)(t-\tau)}dt\,,$$

so that α is merely added to γ.

[10] C. E. Van Orstrand, "On the Empirical Representation of Certain Production Curves," *Journal of the Washington Academy of Sciences*, XV, (1925), 19.

Quite a different kind of levy is represented by the "severance tax."[11] Such a tax, of so much per unit of material extracted from the mine, tends to conservation. The ordinary theory of monopoly of an inexhaustible commodity suggests that the incidence of such a tax is divided between monopolist and consumer, equally in the case of a linear demand function. However, for an exhaustible supply the division is in a different proportion, varying with time and the supply remaining. Indeed, the imposition of the tax will lead eventually to an actually lower price than as if there had been no tax.

Consider the linear demand function

$$p = a - \beta q$$

and, for simplicity, no cost of production. The rate of net profit, after paying a tax v per unit extracted, will be

$$(p - v)q = (a - v)q - \beta q^2 .$$

As in § 4, the derivative increases as compound interest:

$$a - v - 2\beta q = \lambda e^{\gamma t} .$$

Since ultimately $q = 0$ and $t = T$, we obtain

$$a - v = \lambda e^{\gamma T} ,$$

[11] A variant is an ad valorem tax. A great deal of information and discussion concerning these taxes is contained in the biennial *Report of the Minnesota State Tax Commission*, 1928, (St. Paul). From p. 111 of this report it appears that Alabama since 1927 has had a severance tax of 2½ cents a ton on coal, 4½ cents a ton on iron ore, and 3 per cent on quarry products; Montana taxes coal extracted at 5 cents per ton; Arkansas imposes a tax of 2½ per cent on the gross value of all natural resources except coal and timber, 1 per cent on coal, and 7 cents per 1,000 board feet on timber. Minnesota taxes iron ore extracted at 6 per cent on value minus cost of labor and materials used in mining, and also assesses ore lands at a higher rate than other property for the general property tax. These taxes are not based entirely on the conservation idea, but aim also at taxing persons outside the state, or "retaining for the state its natural heritage." Since Minnesota produces about two-thirds of the iron ore of the United States, the outside incidence is doubtless accomplished. Mexican petroleum taxes have the same object. The Minnesota Commission believes that prospecting for ore has virtually ceased on account of the high taxes.

whence eliminating λ and solving for q,

$$q=[1-e^{\gamma(t-T)}](a-v)/2\beta\,.$$

The time of exhaustion T is related to the amount originally in the mine through the equation

$$a=\int_0^T q\,dt=(\gamma T+e^{-\gamma T}-1)(a-v)/2\beta\gamma\,;$$

whence

$$dT=\frac{2\beta a\,dv}{(a-v)^2\,(1-e^{-\gamma T})}\,,$$

showing how much of an increase in time of exploitation is likely to result from the imposition of a small severance tax. The effect upon the rate of production at time t is

$$\begin{aligned}dq&=\frac{\partial q}{\partial v}\,dv+\frac{\partial q}{\partial T}\,dT\\&=dv\{-1+e^{\gamma(t-T)}[1+2\beta\gamma a/(a-v)\,(1-e^{-\gamma T})]\}/2\beta\,.\end{aligned}$$

From the form of the demand function it follows that the increase in price at time t is

$$dp=-\beta dq=dv\{\tfrac{1}{2}-e^{\gamma(t-T)}[\tfrac{1}{2}+\beta\gamma a/(a-v)\,(1-e^{-\gamma T})]\}\,.$$

If a is very large, then so is T; the expression in curly brackets will, for moderate values of t, differ infinitesimally from $\frac{1}{2}$, reducing to the case of monopoly with unlimited supplies. However, dp will always be less than $\frac{1}{2}dv$ and, as exhaustion approaches, will decline and become negative. Finally, when $t=T$, the price of the tax-paid articles to buyers is lower by

$$\beta\gamma av/(a-v)(1-e^{-\gamma T})$$

than the ultimate price if there had been no tax. The price will, nevertheless, be so high that very little of the commodity will be bought.

A tax on a monopolist which will lead him to reduce his prices is reminiscent of Edgeworth's paradox of a tax on first-class railway tickets which makes the monopolistic (and unregulated) owner's most profitable course the reduction of the prices both of first- and of third-class tickets, besides paying the tax himself.[12] The case of a mine is, however, of a distinct species from Edgeworth's, and cannot be assimilated to it by treating ore extracted at different times as different commodities. Indeed, in the simple case of mine economics which we are now considering, the demands at different times are not correlated; supplies put upon the market now and in the future neither complement nor compete with each other. Correlated demand of a particular type was, on the other hand, an essential feature of Edgeworth's phenomenon.

The ultimate lowering of price and the extension of the life of a mine as a result of a severance tax are not peculiar to the linear demand function, but hold similarly for any declining demand function $p(q)$ whose slope is always finite. This general proposition does not depend upon the tax being small.

The conclusion reached in the linear case that the division of the incidence of the tax is more favorable to the consumer than for an inexhaustible supply is probably true in general; at least this is indicated by an examination of a number of demand curves. However, the general proposition seems very difficult to prove.

Since the severance tax postpones exhaustion, falls in considerable part on the monopolist, and leads ultimately to an actual lowering of price, it would seem to be a good tax. It is particularly to be commended if the monopolist is regarded as *unfairly* possessed of his property, and there is no other feasible means of taking away from him so great a portion of it as the severance tax will yield. However, the total wealth of the community may be diminished rather than increased by such a tax. Considering as in § 3 the integral u of the prices p which buyers are willing to pay for quantities below that actually put on the market, and

[12] *Economic Journal*, VII (1897), 231, and various passages in Edgeworth's *Papers Relating to Political Economy*.

the time-integral U of values of u discounted for interest, we have in the case of linear demand just discussed,

$$u=\int_0^q (a-\beta q)dq=aq-\tfrac{1}{2}\beta q^2 .$$

If we were considering the portion of this social benefit which inures to consumers, we should have to subtract the portion pq which they pay to the monopolist, an amount from which he would have to subtract the tax, which benefits the state. But the sum of all these benefits is u, which is affected by the tax only as this affects the rate of production q.

If, for simplicity, we measure time in such units that $\gamma=1$, the rate of production determined earlier in this section becomes

$$q=(1-e^{t-T})(a-v)/2\beta .$$

Substituting this in the expression for u and the result in U, we obtain

$$U=\int_0^T ue^{-t}\,dt$$
$$=(a-v)[4a(1-e^{-T}-Te^{-T})-(a-v)(1-2Te^{-T}-e^{-2T})]/8\beta .$$

We differentiate U and then, to examine the effect of a small tax, put $v=0$. The results simplify to:

$$\frac{\partial U}{\partial v}=-(1-e^{-T})^2a/4\beta$$

$$\frac{\partial U}{\partial T}=[(T+1)e^{-T}-e^{-2T}]a^2/4\beta .$$

From the amount initially in the mine,

$$a=(T+e^{-T}-1)/2\beta ,$$

we obtain, as on page 166,

$$\frac{dT}{dv}=\frac{2\beta a}{a^2(1-e^{-T})} ,$$

when $v=0$. Substituting here the preceding value for a we find, after simplification,

$$\frac{dU}{dv}=\frac{\partial U}{\partial v}+\frac{\partial U}{\partial T}\frac{dT}{dv}=-\frac{a}{4\beta}\frac{e^T+e^{-T}-2-T^2}{e^T-1}.$$

The numerator of the last fraction may be expanded in a convergent series of powers of T in which all the terms are positive. Hence dU/dv is negative.

Thus a small tax on a monopolized resource will diminish its total social value, at least if the demand function is linear. Whether this is true for demand functions in general is an unsolved problem.

We have here supposed the tax v to be constant, permanent, and fully foreseen. Since an unforeseen tax will have unforeseen results, we can scarcely build up a general theory of such taxes. However, any tax of amount varying with time in a manner definitely fixed upon in advance will have predictable results. In this connection, an interesting problem is to fix upon a schedule of taxation v, which may involve the rate of production q and the cumulated production x, as well as the time, such that, when the monopolist then chooses his schedule of production to maximize his profit, the social value U will be greater than as if any other tax schedule had been adopted. This leads to a problem of Lagrange type in the calculus of variations, one end-point being variable. Putting $q=dx/dt$ and

$$J=\int_0^T f(x, q, v, t)\,dt\,, \qquad U=\int_0^T F(x, q, t)\,dt\,,$$

the problem is to choose v, subject to the differential relation

$$\frac{\partial f}{\partial x}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial x}-\frac{d}{dt}\left(\frac{\partial f}{\partial q}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial q}\right)=0\,,$$

so that U will be a maximum. In general, of course, a still greater value of U would be obtainable, at least in theory, by public ownership and operation.

14. MINE INCOME AND DEPLETION

With income taxes we are not concerned except for the determination of the amount of the income from a mine. The problem of allowance for depletion has been a perplexing one. It has been said that if the value of ore removed from the ground could be claimed as a deduction from income, then a mining company having no income except from the sale of ore could escape payment of income tax entirely. The fallacy of this contention may be examined by considering the value of the mine at time t,

$$J(t)=\int^{T} pqe^{-\gamma(\tau-t)}d\tau\,.$$

In this integral p and q have the values corresponding to the time τ, later than t, assigned by whatever production schedule has been adopted, whether this results from competition, from a desire to maximize monopoly profit, or from any other set of conditions. The net income consists of the return from sales of material removed (cost of production and selling having as usual been deducted), minus the decrease in the value of the mine. It therefore equals, per unit of time,

$$pq+dJ/dt\,;$$

and from the expression for J, this is exactly γJ. In other words, any particular production schedule fixes the value of the mine at such a figure that the income at any time, after allowing for depletion, is exactly equal to the interest on the value of the investment at that time.

But, although the rate of decline in value of a mine seems a logical quantity to define as depletion and to deduct from income, such is not the practice of income-tax administrations, at least in the United States. The value of the property upon acquisition, or on March 1, 1913, a date shortly before the inauguration of the tax, if acquired before that time, is taken as a basis and divided by the number of units of material estimated to have been in the ground at that time. The resulting "unit of depletion," an amount of money, is multiplied by the number of tons, pounds,

or ounces of material removed in a year to give the depletion for that year. The total of depletion allowances must not exceed the original value of the property.

The differences between the two methods of calculating depletion arise from the uncertainties of valuation and of forecasting price, demand, production, costs, interest rates, and amount of material remaining. If the theoretical method were applied, a year in which the mine failed to operate would still be set down as yielding an income equal to the interest on the investment value. This seems anomalous only because of another defect, from the theoretical standpoint, in income-tax laws: the non-taxation of increase in value of a property until the sale of the property. During a year of idleness, if foreseen, the value of the property is actually increasing, for the idle year has been considered in fixing the value at the beginning of the year.

An amendment made to the United States federal income-tax law in 1918 provides that the valuation upon which depletion is calculated may under certain circumstances be taken, not as the value of the property when acquired or in 1913, but the higher value which it later took when its mineral content was discovered. This provision has the effect of materially increasing depletion allowances, and so of reducing tax payments. The sudden increase in value when the mineral is discovered might well be regarded as taxable income, but is not so regarded by the law unless the property is immediately sold. The framers of the statute seem indeed, according to its language, to have considered this increase in value a reward for the efforts and risks of prospecting, which would suggest that it is of the nature of income, a reasonable position. However, the object of the amendment is to treat this increment as pre-existing capital value, to be returned to the owner by sale of the mineral. The amendment appears to be inconsistent and quite too generous to the owners particularly affected.

15. DUOPOLY

Intermediate between monopoly and perfect competition, and more closely related than either to the real economic world, is the condition in which there are a few competing sellers. In a for-

mer paper[13] this situation was discussed for the static case, with special reference to a factor usually ignored, the existence with reference to each seller of groups of buyers who have a special advantage in dealing with him in spite of possible lower prices elsewhere. More than one price in the same market is then possible, and with a sort of quasi-stability which sets a lower limit to prices, as well as the known upper limit of monopoly price.

For exhaustible resources the corresponding problems of competition among a small number of entrepreneurs may be studied in the first instance by means of the jointly stationary values of the several integrals representing discounted profits. We need not confine ourselves, as we have done for convenience in dealing with monopoly, to a single mine for each competitor. Let there be m competitors, and let the one numbered i control n_i mines, whose production rates and initial contents we shall denote by $q_{i1}, \ldots, q_{in_i}$, and $a_{i1}, \ldots, a_{in_i}$, respectively. The demand functions will be intercorrelated, both among the mines owned by each competitor and between the mines of different concerns. Consequently the m integrals J_i representing the discounted profits will involve in their integrands f_i all the q_{ij}, as well as some at least of the cumulated productions $x_{ij} = \int q_{ij}\, dt$. If the i^{th} owner wishes to make his profit a maximum, assuming the production rates of the others to have been fixed upon, he will adjust his n_i production rates so that

$$\frac{\partial f_i}{\partial x_{ij}} - \frac{d}{dt}\frac{\partial f_i}{\partial q_{ij}} = 0. \qquad (j = 1, 2, \ldots, n_i)$$

Continuing the analogy with the static case, we are to imagine that the other competitors, hearing of his plans, do likewise, altering their schedules to conform to equations resembling those above. When the i^{th} owner learns of their changed plans, he will in turn readjust. The only possible final equilibrium with a settled schedule of production for each mine will be determined by the solution of the set of differential equations of this type, which are exactly as numerous as the mines, and therefore as the vari-

[13] Harold Hotelling, "Stability in Competition," *Economic Journal*, XLI (March, 1929), 41.

ables to be determined. All this is a direct generalization of the case of inexhaustible supplies. But we shall show that the solution tends to overstate the production rates and understate the prices of competing mines.

Doubts in plenty have been cast upon the result in the simpler case, and the reasons which can there be adduced in favor of the solution are even more painfully inadequate when the supplies are of limited amount. The chief difficulty with the problem of a small number of sellers consists in the fact that each, in modifying his conduct in accordance with what he thinks the others are going to do, may or may not take account of the effect upon their prices and policies of his own prospective acts. There is an "equilibrium point," such that neither of two sellers can, by changing his price, increase his rate of profit while the other's price remains unchanged. However, if one seller increases his price moderately, thus making some immediate sacrifice, the other will find his most profitable course to lie in increasing his own price; and then, if the original increase is not too great, both will obtain higher profits than at "equilibrium." But that the tendency to cut prices below the equilibrium is less important than has been supposed is shown in the article just referred to.

With an exhaustible supply, and therefore with less to lose by a temporary reduction in sales, a seller will be particularly inclined to experiment by raising his price above the theoretical level in the hope that his competitors will also increase their prices. For the loss of business incurred while waiting for them to do so he can in this case take comfort, not merely in the prospect of approximating his old sales at the higher price in the near future but also in the fact that he is conserving his supplies for a time when general exhaustion will be nearer and even the theoretical price will be higher. Thus a general condition may be expected of higher prices and lower rates of production than are given by the solution of the simultaneous characteristic equations.

For complementary products, such as iron and coal, the situation is in some ways reversed. Edgeworth in his *Papers Relating to Political Economy* points out that when two complementary goods are separately monopolized the consumer is worse off than

if both were under the control of the same monopolist. This assumes the equilibrium solution to hold. The tentative deviations from equilibrium made in order to influence the other party may now be in either direction, according as the nature of the demand function and other conditions make it more profitable to move toward the lower prices and larger sales characterizing the maximum joint profit, or to raise one's price in an effort to force one's rival to lower his in order to maintain sales. When the supplies of the complementary goods are exhaustible, the same indeterminateness exists.

A very different problem of duopoly involving the calculus of variations has been studied by C. F. Roos,[14] who finds[15] that the respective profits take true maximum values. However, as in the static case, no definitively stable equilibrium is insured by the fact that each profit is a maximum when the other is considered fixed, since the acts of one competitor affect those of the other. The calculus of variations is used by Roos and G. C. Evans[16] to deal with cost and demand functions involving the rate of change of price as well as the price. Such functions we have for concreteness and simplicity avoided, but if they should prove to be of importance in mine economics the foregoing treatment can readily be extended to them. (Cf. § 12.) Evans and Roos are not concerned with exhaustible assets, and assume that at any time all competitors sell at the same price.

The problems of exhaustible resources involve the time in another way besides bringing on exhaustion and higher prices, namely, as bringing increased information, both as to the physical extent and condition of the resource and as to the economic phenomena attending its extraction and sale. In the most elementary discussions of exchange, as in bartering nuts for apples, as well as in discussions of duopoly, a time element is always introduced to show a gradual approach to equilibrium or a breaking

[14] "A Mathematical Theory of Competition," *American Journal of Mathematics*, XLVII (1925), 163.

[15] "Generalized Lagrange Problems in the Calculus of Variations," *Transactions of the American Mathematical Society*, XXX (1927), 360.

[16] *A Mathematical Introduction to Economics* (McGraw-Hill, 1930).

away from it. Such time effects are equally or in even greater measure involved in exploiting irreplaceable assets, entangling with the secular tendencies peculiar to this class of goods. With duopoly in the sale of exhaustible resources the possibilities of bargaining, bluff, and bluster become remarkably intricate.

The periodic price wars which break the monotony of gasoline prices on the American Pacific Coast are an interesting phenomenon. Along most of the fifteen-hundred-mile strip west of the summits of the Sierras a few large companies dominate the oil business. In the southern California oil fields, however, numerous small concerns sell gasoline at cut prices. Cheap gasoline is for the most part not distilled from oil but is filtered from natural gas, and may be of slightly inferior quality; nevertheless, it is an acceptable motor fuel. The extreme mobility of purchasers of gasoline reduces to a minimum the element of gradualness in the shift of demand from seller to seller with change of price. Ordinarily, the price outside of southern California is held steady by agreement among the five or six major companies, being fixed in each of several large areas according to distance from the oil fields. But every year or two a price war occurs, in which prices go down day by day to extremely low levels, sometimes almost to the point of giving away gasoline, and certainly below the cost of distribution. From a normal price of 20 to 23 cents a gallon the price sometimes drops to 6 or 7 cents, including the tax of 3 cents. Peace is made and the old high price restored after a few weeks of universal joy-riding and storage in every available container, even in bath tubs. The interesting thing is the slowness of the spread of these contests, which usually begin in southern California. The companies fight each other violently there, and a few weeks later in northern California, while in some cases maintaining full prices in Oregon and Washington. These affrays give an example of the instability of competition when variations of price with location as well as time complicate commerce in an exhaustible asset.

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PROFESSOR FISHER'S INTEREST THEORY: A CASE IN POINT

THE present paper was "provoked" by Professor Fisher's restatement of his interest theory,[1] but is not a review of his book or even properly an article on the same subject. The reader is warned at the outset that it is a "rethreshing of the old straw" not only of interest theory in general, but worse, of price theory in general. It might be called a "review" of his full title—*The Theory of Interest as Determined by Impatience To Spend Income and Opportunity To Invest It*—with especial reference to the two words "determined" and "and." But the real target is the more general proposition that any price is "determined" by subjective "and" objective factors (the terms repeatedly used by Professor Fisher) or, in more common parlance, utility and cost of production. Two contentions are to be argued: first, that the prices of ordinary, typical commodities are practically "determined" by objective factors, cost or productive conditions, subjective (utility) factors typically exerting a large influence on supply but an insignificant one on price; and, second, that the interest rate is an extreme example of this principle, regarding which it is especially true and demonstrable that the subjective factors ("impatience," "time-preference") are in a peculiar degree purely passive. Thus the argument is directed primarily against the generally accepted "eclectic" or "equilibrium" statement of general price theory, according to which demand and supply are "like the two blades of a pair of scissors." Professor Fisher's interest theory is useful as "a case in point" because of the intrinsic importance of its own problem and of its treatment at the author's hands, and because of the generally inadequate treatment of this aspect of interest as a case of price determination. Professor Fisher himself does not adopt the view of interest as a price like other prices or indicate the parallelism or possible concrete

[1] Irving Fisher, *The Theory of Interest*. New York: Macmillan, 1930. (A revision and re-writing of his *The Rate of Interest*. New York: Macmillan, 1907.)

differences between the mode of its fixation and that of ordinary prices; nor does he employ the usual price analysis with supply and demand curves or functions.[2] If he had given a little consideration to the mode of variation of amounts supplied and demanded in relation to price (interest rate) variations, he would probably have modified his insistence on the complete co-ordinateness of the two sets of conditions in determining the price, which is the main theme of his revised work (pp. 177, 182, 495, 496, etc.). It is the main task of the present paper to put the basic facts of the workings of subjective and objective conditions in price determination and their relations to supply and demand in a form which avoids some errors in conventional presentations and makes them less open to misinterpretation, if not simpler.

I. THE INTEREST RATE AS A PRICE

In order to discuss interest theory, either on its own account or as "a case in point," it is necessary to be clear in regard to the precise nature of the phenomenon. In general, I shall accept the conceptions formulated with admirable clearness and logic by Professor Fisher. His theory of wealth as purely anticipated real income in consumption satisfactions I regard as a pernicious half-truth,[3] but it is a very convenient rationalization for the sake of

[2] Fisher says (p. 61): "The rate of interest expresses a price in the exchange between present and future goods," and also refers to the problem as one of the "supply and demand of future income" (p. 32).

[3] It seems to me indisputable in fact that people desire wealth for many reasons, of which the guaranty of the future delivery of groceries or other consumable services is sometimes the main and sometimes a quite minor consideration. It is desired for the same reasons a head-hunting hero desires a goodly collection of skulls; it is power, a source of prestige, a counter in the game, an article of fashion, and perhaps a mere something to be "collected." It is wanted to use, but also just to have, to get more, in order to get still more. Many of the other bases of the desire to own wealth have some connection with the fact that it yields "income" in one sort or another, but the connection should not be confused with identity.

The position to be taken on this point depends on the problem in hand and the point of view; it is a very different matter for an accountant, a drafter of tax legislation, a "scientist" interested in understanding and explaining human behavior, and a social philosopher attempting to criticize the effects of "freedom of contract" on the evolution of culture and life.

In the historical literature of economics, one of the outstanding ambiguities relates to the nature of the primary economic motive, which is now assumed to be con-

simplicity and perhaps defensible for the purposes of abstract price theory frankly stated in terms of the assumption of economic rationality. Anyhow, it will be accepted "for the purpose of the argument." Another point should be mentioned. In a perfect capital market and under "static conditions," with risk eliminated and known investment opportunities open, there would be only one rate of interest, regardless of the period of the loan, the shape of the income stream involved, or other factors (contrast Fisher, p. 206). All loans might be made without maturity, since they would be instantly salable at a perfectly definite value. Stress should be laid on the arithmetical equivalence of all forms of interest under these "ideal conditions" necessary for abstract theory. Hence the exchange between income streams of different shape, while more general and accurate, is less usable than the notion of exchange of a given sum of wealth "now" for either a perpetual income stream of given magnitude or a given amount receivable at a specified future date. In effect, any produced durable good is the embodiment of a perpetual income, since it must replace itself before it wears out, with an excess, or it will never be constructed (in a world in which investment opportunities are open); only the excess after full provision for maintenance and replacement is really income.[4] The first essential fact is that stated in the citation above (p. 177, n. 2), that the interest rate is a price in the exchange between present and future goods. It is then to be discussed and explained in the same general terms as the exchange ratio between any two commodities, such as wheat and cotton, or deer and beaver (in the example of Adam Smith).

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sumption and now accumulation for its own sake. Both certainly play a part, though neither is really ultimate. Perhaps the economist is justified in treating them as ultimates for his purposes; but he certainly ought to keep them apart. Certainly their interrelations are not to be disposed of—are indeed grossly misrepresented—by setting up any form of the capitalization formula as a transformation function between them.

[4] In a frictionless market, every piece of "wealth" must yield at every moment an income in some form (service, product, increase in its own value) which on a perpetual basis is at the same rate on its sale price as that of any other. An owner behaving rationally will exchange anything which yields less for any which yields more, until revaluations bring all to equality. The relation between this process and the production of new wealth is the central problem of the interest rate.

The second absolutely fundamental fact, in connection with the occurrence of interest as it occurs in the world, is that these two types of goods—namely, things rendering all their satisfactions "now" and those rendering them in a perpetual stream—are both being constantly produced in large volume. That is, they are like most other possible pairs of commodities or types of commodities which come into the market and receive prices (not like "old masters" and a few other exceptions) in that producers are constantly choosing which they shall produce. As long as this condition holds, the ratio of exchange between the two commodities or classes of commodity must be such that the amounts which exchange against each other are the amounts between which producers have an open choice in production on equal terms.

In the famous example, if it costs two days' labor to kill a beaver and one day to kill a deer, then productive adjustment will establish an equilibrium such that one beaver exchanges for two deer. So far is this "cost theory" from being in contradiction with or even divergent from "utility theory" that the latter is absolutely presupposed in the argument. The reason this exchange ratio must prevail is that if one beaver is exchanging for more than two deer, anyone who wants deer will hunt beaver and exchange for deer instead of hunting deer, and conversely; and that when more beaver and fewer deer are brought to the market the price of the former will fall and that of the latter rise until they correspond with the cost ratio, and conversely. It makes no difference what consumers think of the relative utilities of deer and beaver. The only question is that of whether, as more beaver and fewer deer are killed, the relative costs will remain unchanged. This question will be taken up presently. The mystery of classical price theory is that the writers never raised the question of the process or "implementation" of the adjustment (which would have brought out the utility principle) nor saw that it makes no difference what is the nature or content of "cost" or whether there is any cost, as long as producers have their choice between definite amounts,[5] and that they never explicitly asked

[5] Senior may be said to have recognized this, just as he rather clearly stated the utility principle. But in the former case his statement involves a fallacy.

what would happen to the cost ratio under variation in output. Before taking up this question it is well to clarify some points about the situation in which interest occurs. In much of this no important disagreement with Professor Fisher is involved.

In the first place, as to consumption or "spendthrift" borrowing. It might be admissible as an approximation to say simply that borrowing for productive purposes is of so much greater volume as to "dominate" the rate. But it is not accurate, and the case for neglecting the consumption loan is in fact much stronger. In a world where new net saving and investment continually take place, loans for consumption are simply a deduction from the supply offered in the market for productive loans, and influence the rate in the latter only as a deduction from supply. As the further course of the argument will show, this means that they do not influence it at all, or not to any significant or conceivably discoverable extent, since on general grounds no possible variation in supply can appreciably affect the rate.

Less simple is the case of another type of loan, namely, those made in connection with the purchase and sale of productive goods already existing (in contrast with the creation of such goods). Suppose that A borrows from B to buy from C a farm, an automobile, or a patent right. This transaction is not final, not the complete event. Up to this point certain parties have simply exchanged places, without affecting the ultimate demand or supply of anything. The final effect of the transaction depends on how B got the money and what C does with it. It may be new money put into circulation for the first time, in which case the main consequence would normally be to raise the price level, but "forced saving" will also be involved if C uses it productively. If it is money in regular circulation and C uses it for consumption purposes, the whole transaction becomes a consumption loan; or if he uses it for productive purposes, a productive loan.

Finally, it is to be noted that the actual purchase and sale or direct barter of present wealth for existing income-bearing goods is similar to the case just noticed and does not affect the interest rate. As has been pointed out often enough, the value of existing capital goods is the effect of the interest rate and not directly a

cause. It does not seem to be realized by all who carefully stress this fact that it is true only if the "productivity" theory of interest is true! Under a "time-preference" theory the rate would be determined in just such transactions, and under the "eclectic" theory of Professor Fisher they would play their equal part along with productivity, or "investment opportunity" as he prefers to call it.[6] As it is the main theme of this paper to show, exchange ratios are typically "determined" by conditions outside the exchange itself, namely, in the conditions of production, and this is true in a peculiarly complete degree in the case of the ratio of exchange between present goods and durable goods. In a world where goods result from production in which producers have a choice on equal terms between definite amounts of any two varieties or classes, the production of a definite amount of either can be "purchased" by giving up the corresponding amount of the other. No person behaving intelligently is going to give up more to "purchase" a good in exchange than the amount for which it can be "purchased" by shifting production, and since the rule works both ways, he cannot get it for less, and this fixes the only possible exchange ratio. The relation between durable goods and present goods differs from that typical for any pair of present goods in the more extreme fixity of the production ratio (its independence of the amounts produced) and the resulting completeness of its dominance over the forces working in the exchange itself.

II. THE MEANING OF COST OF PRODUCTION AND THE NATURE OF ITS INFLUENCE ON PRICE

The mystery in connection with value theory is that it has taken so long to put a few simple and familiar facts together

[6] The reasons given for the preference are that "capital value does not produce income value" (pp. 55, 150) and that the productivity theory "attributes the value of manufactured things to the cost of production, instead of to their discounted future services." Both statements are in different ways plays on the word "value." Capital goods produce things or services which have more value than that embodied in the capital goods themselves in their own production. And while it is correct to say that commodities derive their "value," in the general, human sense, from their power to render service, their relative value or price is typically determined to a virtually exclusive degree by their relative cost, as indicated above and as will presently be shown more fully.

coherently into a "system," or reasoned statement of the workings of the processes in relation to each other. This, in the writer's opinion, is due, in large part at least, to a fundamental confusion of categories. Both the terms "value" and "cost" are highly ambiguous. In the first place, each is used in three different senses, belonging to three main universes of discourse. Each may be, and habitually is, viewed as a physical magnitude, a quantity of some measurable thing; also, alternatively, as a psychical magnitude, a feeling in a mind, which has in some way and degree a quantitative character but is not actually measurable; in the third place, each may and frequently does refer to a kind of moral or normative entity, the quantitative nature of which is still more vague and dubious. Physical value is price; physical cost, days of labor in the visible sense, or days of use of some other agency, or consumption of some physical material. Psychical value is usually called utility and defined as conscious satisfaction or pleasure, psychical cost being the corresponding disutility or pain. Such questions as whether the deprivation of pleasure is a pain, and release from pain a pleasure, whether work is painful, and similarly as to waiting, simply start arguments as futile as they are endless, since they have no significance for the theory of price determination anyway, as the next few pages will show.[7] In the third meaning value and cost have to do with what is assumed to be the object of social policy. (Ethics in the idealistic or religious sense would be a still further meaning, and is not in question here.) Judgments regarding policy are generally explicitly, not to say noisily, excluded from economic discussion, but are consciously or unconsciously introduced and perhaps get their equal share of the actual usage. Otherwise stated, the three contrasting viewpoints relate to (1) what people actually, visibly do; (2) what they try to do or are assumed in utilitarian theory to try to do, i.e.,

[7] Professor Fisher is especially insistent that labor is a cost and waiting not, evidently because he thinks that the simpler assumption for accounting. Others take the opposite view. It would seem logical to list as a cost in the accounting sense anything that has to be paid for, and waiting is such an item. Fisher and economists generally seem to be confused by the dual rôle of time as a dimension in the use of goods and as chronological or elapsed time. The point will come up again (see below, p. 198, n. 14).

what they "would" do if they behaved with complete rationality; and, finally, (3) what they "should" do, or would do if they behave rationally under "sanctions" established by an intelligent central authority representing the interests of all.

This is not the end of the ambiguity. On the value side price may mean money, or a quantity of some other commodity. With cost there are a number of additional meanings. Physical cost may, like physical value, be stated in money. Or it may be interpreted as a quantity of another consumption good which might have been had for the same productive expenditure. Finally, there is a very important difference in the apparent meaning as well as magnitude of cost according as one thinks of the cost of an entire "income" or of a small portion or increment of "economic" income. This last confusion has played as much havoc with sound thinking as anything in the history of economics; it will be noticed again in a moment (p. 185). If one is uncritical enough, a tendency to correspondence, direct or inverse, between practically any two of these "magnitudes" may be discovered and erected into a causal relation, in one direction or the other; or the direction may be a matter of opinion. On the other hand, it is uncertain just what causality in any such case means or involves. The Ricardian school thought of explanation as the finding of another magnitude to which "value" "had to be" "equal," and never worked out in terms of recognized utilitarian behavior principles the process by which the equality is established and maintained, or even the conditions under which it would exist. Price was seized on as the primary fact to be explained—which choice itself must be explained historically, since there are several other magnitudes determined along with prices which are more important! Psychical value or utility was rejected as an explanation because of empirical non-correspondence (water versus diamonds, iron versus gold). Smith really held a money-cost theory, but Ricardo and his followers turned to psychical cost. The utility school substituted psychical value. Marshall makes "utility" and "cost of production" co-ordinate (illustration of the scissors) without indicating what he means by cost in this connection. Only recently has the view herein treated as directly relevant and correct made head-

way, under the name of "opportunity cost" or "alternative cost."

Now to any mind which, looking out upon a world in which prices exist, is moved by curiosity, and attempts to explain them, certain facts are evident beyond dispute. In order that any exchange may take place, at least two persons must make a comparison between successive increments of one commodity parted with and of the other obtained, discovering such a change in the relative importance of the two kinds of increments as does bring the process to a stop (diminishing utility). The condition at the point where exchange ceases is described by saying that the use-value or utility (desiredness) of a small increment of either commodity is "equal," to both parties, to that small increment of the other commodity for which it was exchanged at the stopping-point. Whether there is ever a real comparison of separately existing utility magnitudes, whether diminishing utility is absolute or in essence relative, we need not here stop to argue. It is permissible to speak of indifference as due to equality, so long as the magnitudes are not brought into quantitative ratio when they are unequal; the latter, though commonly done, is a violation of the utility principle itself as well as contrary to obvious facts (cf. p. 192, n. 11).

In a market where goods are continuously exchanged, flowing through the market from production to consumption, an approximately uniform price is quickly established, at which all exchanges take place. Then the "relative marginal utilities," accurately the relative increments of different commodities having equal utility, are the same to all the exchangers in the market. The absolute utility associated with any commodity-increment to any person, or its comparative magnitude for different persons, never comes in question in the theoretical explanation of the phenomena, however important it may be for questions of ethics and of social policy. It is a curious fact that these simple relations baffled many of the "best minds" from the dawn of the science and are not yet clearly stated in many general expositions. Continuous exchange is, of course, associated in this world with continuous production. Production, as already noted, typically goes

forward under the condition that producers have a choice between different lines or commodities. The terms of this choice bring up the general conception of cost. The most interesting and important characteristic of this concept is its endless ambiguity. As already noted, its actual rôle in economic theory is that of obscuring the really fundamental fact of alternatives of choice in production; it is the existence of terms of choice which affects price, the content or even existence of cost being theoretically immaterial. But some understanding of the conditions determining alternatives in production in the actual world is useful in shedding light on the character of the terms of choice themselves.

To the question what anything costs, we give many kinds of answer.[8] In the field of economics proper, two propositions may be put forward as true and important in untangling truth and error in the theory of price. The first is that the cost in general, to an individual or society, of securing an income in the sense of his (or its) total aggregate of satisfactions does consist of the "pain" (or sacrifice of "pleasure," if there is any difference, or both) involved in "production" in an exhaustive sense. The use of material instruments is not a cost, except as these in turn cost pain, for under the assumptions of rational utilitarianism (here accepted for the purpose of the argument) they have no other use or meaning than to yield an income, in the inclusive sense of that term. The other principle is that with reference to a money income or economic income as a part of the total flow of satisfactions, if the aggregate of this part is of fixed magnitude and the actual range of choice under consideration involves only its detailed composition, then the real "cost" of any particular small increment of the money income is the sacrifice of the competing increment which was actually given up in order to get the one in question. Confusion of these two viewpoints is undoubtedly at the bottom of much of the ambiguity and irrelevance in the controversial literature of price theory.

[8] We must pass over the interesting question of costs in the moral sphere. The Old Testament and the New give the most contradictory possible views of the rewards of virtue; compare the Twenty-third Psalm with the Sermon on the Mount—or still better, with the conception of perfect happiness of St. Francis of Assisi.

But the trouble by no means ends here. With the intervention of money as the symbol and embodiment of "commodity," every good and service not specifically in question in its individual character in any case is thought of as an amount of money. So the consumer thinks of the "cost" of every item in his real income as a money expenditure, even though he will probably admit "if pressed" that it is "really" the other use that he would have made of the money. And, worse still for logic, under an enterprise system of organization the efforts and outlays involved in production are marketable goods with definite pecuniary values, so that the "cost of production" also comes to be thought of as a sum of money. And here we are so far removed from the actual final alternatives between which choice is made, and have brought so many persons into the sequence, that it is by no means certain that any of them will see at a glance that what any item of real income really cost was that, through the use in production of some agency or resource by some one in one way instead of by someone else in another, some other item of real income was sacrificed in order that this item might come into existence. Neither common sense nor business accounting are concerned to follow the facts to the final real forking of the roads.

Even those who start to follow the sequence through commonly lose courage or suffer a lapse of attention at the last step and seize upon the expenditure of productive service as the real cost, forgetting that these signify nothing in themselves, but only through the use to be made of them. For reasons which a little reflection will make evident enough, this is particularly likely to happen in connection with that element in the production cost representing direct human psychical effort and sacrifice. Yet it need only be pointed out that under the conditions of modern economic life the laborer's efforts are practically like other outlays; if they are not fixed in total amount, at least the choice between more and less labor is independent of that between different directions of its expenditure. Besides, it is overlooked that the physical outlays are confronted with essentially the same alternative of choice as that between work and "leisure" open to the laborer, namely, they can also "go and play" and in considerable measure actually

do so, through being used by their owners to yield direct satisfactions instead of products for the market. Thus, while they do not represent a cost of the total income of satisfactions of an individual or of society, they do mediate a cost of income in the sense of a sum of money or money value. The distinction that pain is a cost and the use of physical materials and agencies not a cost holds logically only for a completely inclusive view of income. The relative magnitude and importance of leisure as compared with the non-monetary uses of the materials and forces of nature may be enough greater to call for notice, but it should not be exaggerated into a difference in kind.

There is still another source of confusion, interacting with and aggravating the others as well as supplementing them, and perhaps most important of all. It has been emphasized above that the nature and content of cost really has nothing to do with the general theory of price, the only essential fact being that the goods priced are produced (or suffered to come into being) by people having a choice between producing definite amounts of different commodities, which amounts must be equivalent in exchange (made so by the adjustment of production) if consumers and producers behave intelligently. But our minds do not generally work in terms of such general principles, or at least not correctly. Students instead of seeing this elementary fact have rested their eyes on the physical basis of the productive choice, which obviously is that the means of production, "efforts and outlays," can be applied to one use or another, and transferred from one use to another. The productive choice is inevitably seen to be determined by the relative value of the alternative results of the employment in different uses of the same[9] productive means, efforts, or outlays. But looking at the concrete means of production, and at the concrete facts of their allocation to different uses and transference from one to another, inevitably focuses attention on limitations and qualifications. Some of these are really important in the statement of the principles with reference

[9] The emphasis on identity instead of equality needs a little qualification for psychical or sentimental differences between different occupations. These are particularly evident in the case of labor (irksomeness, etc.) but are real for all factors.

to any particular case at a given location and given moment of time, while some are altogether misleading.

Viewing the economic order concretely, as a going concern, one is struck by the impediments to "freedom" in the productive choice. Few of the means of production, human and non-human, can and do respond instantly to the existence of a possibility of realizing a slightly greater value product in another field than that in which they are actually at work, and even those momentarily unemployed do not choose precisely and instantaneously the best among the alternatives open. Most of them must be played upon by a considerable differential for a considerable period of time—though there is also much changing without real reason or in the face of negative advantage; but this, while theoretically an offset, increases the difficulty of seeing the theory through the facts. Worst of all—and this is the real source of confusion in point—it is rather generally true that the concrete means of production are "specialized" to particular uses and cannot move at all, or can do so only at great apparent loss in effectiveness.

The complication and confusion are enormously multiplied by three features of the productive organization. The first is the mere fact that commodities are produced by the use of many kinds of means of production, working together in complicated ways, so that it is not possible directly and visibly to separate their effects and say how much commodity of what quality and value is actually produced by any one man or property item. The second feature is that the production process is long, and divided into infinitely complex and overcrossing stages (Menger's goods of different "order"). The "means of production" used in making the final good or service next to the consumer are typically (not always, which makes it worse) products of other means of production, which are in turn products, and so on, in a series long in time as well as empirically intricate, going back to productive activities of past generations, even of peoples unknown to history, and carried out under conditions totally different from those obtaining "now." The third feature, superimposed on the foregoing, is the crudity of the pecuniary valuations attached at all the stages,

in terms of which decisions are actually made for the most part. These valuations are inaccurate, subject to inertia and caprice, and based on irrational, backward-looking considerations about as much as on potentialities, which alone are "intelligent" grounds of choice. (Most of the ethical principles affecting economic activity involve more or less conflict between backward-looking and forward-looking attitudes, between "absolutes" of some sort and "rational expediency"; and there is much apparent contradiction affecting choices even when reconciliation of principles is possible.)

In short, at any given time, the concrete means of production are "bound" to particular uses in all degrees, but in very large degree on the whole. But, given time for adjustment, the typical situation is that they wear out or otherwise superannuate after intervals, again of all lengths, and have to be replaced by new units. And in replacing any unit it is typically possible to make instead a unit of a different kind to be employed in a different use, so that by waiting until any piece of equipment wears out, the "productive capacity" "embodied" in it can be transferred to another field, even if the agent in the concrete sense is completely specialized. As the goods of next order are likely also to be partly specialized, and even farther, it may be a long time before complete mobility is accomplished. In the case of human beings the basis of specialization is different; retraining is possible, but the ability to control the production of special adaptabilities is limited, and other factors than efficiency play a part in such control as is practicable. As between human beings and non-human agencies still further factors modify the transformation of productive capacity in either form into the other when the latter would yield product of greater exchange value.

In view of all limitations, the typical situation regarding the production of any particular commodity is that it is made in part by means of production which could be freely moved to other uses, and in part by such as could not be so moved. But the greater the time allowed for change, the greater the freedom to move the latter, without loss, by replacement with others of a different kind. But, again, this freedom would never become com-

plete. Some means of production are permanently specialized to particular commodities or limited fields of production. These facts in themselves would not be so serious, though they mar the simplicity both of adjustments themselves and of their description. But when all these concrete means of production have values, subject to all the irrational factors which do influence pecuniary valuation, and—still worse—are practically replaced by abstract, colorless amounts of money in the thinking of production managers ignorant of the source and meaning of money valuations, then the effects on the control of production do become serious. And since students of economic phenomena are themselves confused by pecuniary thinking, even greater havoc is wrought in the formulation of the theoretical description.

If rational money valuations and rational response to them be postulated, the actual relations and adjustments may be covered by fairly simple general principles. (There might still be long-period contracts in money terms, deference to custom, etc., etc., if the parties would segregate their speculative gains and losses on such contracts, and the costs of being conventional, humane, or lazy from the results of productive activity in the usual sense and would manage the latter rationally.) In the first place, the productive effect of any agency in any use is the increment of effect from an increment of the agency, in the units in which it is actually bought and sold and manipulated, absolutely regardless of whether that agency functions alone in the production process or in co-operation with any others. And conversely, the money cost of any product-increment is the value of the increment of productive service which adds that unit to the total output. Production managers find productive services in the market, with prices beyond their personal control. They hire them at existing prices in the "best" proportions (always substituting a dollar's worth of one for a dollar's worth of another whenever a given increment of product can be more cheaply secured), and carry production of any commodity to the point where the exchange value of its last increment is just equal to its increment of cost. The use of commodities is determined by similar choices (substitutions) on the part of the consumer, who is similarly unaffected

in principle by interconnections between commodities in consumption; he uses the best proportions and in theory carries the total to the point where an increment of consumption is no longer worth more than the increment of leisure or saving which it costs, but actually about as far as he "can." The prices of both commodities and productive services are set by sellers, at the point which will clear the market. General equilibrium is established when, at the same time, (1) every consumer is getting the maximum satisfaction possible in his circumstances, i.e., without preying upon anyone else; (2) the entire output of every commodity is sold and consumed; (3) each productive establishment, using the best proportions, has carried its operations to the no-profit point; and (4) there is no unemployment.

The conditions thus stated in pecuniary terms are valid regardless of time allowed for making changes; they would be established very quickly and maintained continuously throughout any process of readjustment however long or complicated if only price relations were kept rational. "Ultimate" ("static") equilibrium[10] is a different matter; this results when no further readjustments in either consumption or production with accompanying revaluations are advantageous to anyone. In adjustments within any limited time period some transfers of productive agents will be profitable and others not. Those agents which it is not profitable to transfer will simply be valued with reference to the use they are already in, thus establishing the pecuniary equalities stated. It is necessary to note only that means of production which at any time or with reference to any period are not on a margin of indifference as to moving to another use do not mediate a productive choice and consequently do not affect the exchange ratio between any commodities. Such agencies, valued only with reference to a particular use, whether temporarily or permanently, are still valued on the same principle, each increment of service being worth its increment of product (monopoly excluded) in con-

[10] The term "static" is misleading and its correlate, dynamic, positively vicious. It is always necessary to be clear as to what elements are assumed fixed and what variable. The conventional "static state" is roughly indicated by specifying given (1) wants, (2) resources and distribution of ownership, and (3) technology or state of the arts.

sequence of competition among producers of the particular commodity. During any process of movement toward ultimate equilibrium, as well as at the end of it (assuming rational management and pricing), the exchange ratio or relative price of any two commodities must continuously be identical with the relative amounts having the same cost, i.e., the amounts constituting alternative product increments of transferable agencies of production actually on a pecuniary margin of indifference between the two uses. The ratio will of course change during the progressive adjustment, as more agencies come, through superannuation, to the fluid state and the indifference margin. The prices of non-transferable agencies are "rents" in the theoretical sense. As stated above, the price, after making of all transfers of production which are profitable, either ultimately or within any given time period (and after the correct revaluations), must be that described by saying the commodity increments of the same cost exchange for each other. And "cost" means either real cost, as defined, or money cost.

But this does not prove, or assert, that cost "determines" the price. It is even more obvious that the price in any case also conforms to utility, in the same sense, namely, that the amounts of any two commodities equivalent in exchange must be the amounts equivalent in consumption to every individual consumer in the field of competition, whatever his taste or income (i.e., everyone who is on a margin of indifference with respect to units of the two goods—which practically means everyone who consumes both. If dollar's worths of two commodities are not of "equal utility" (as increments) to any such individual, he is by the definitions of the terms not buying and consuming rationally. Sellers on their part set their prices so as to dispose of their entire supplies (streams, not stocks) when the supply itself is at the no-profit point. Since things equal to the same thing are equal, the cost ratio must be constantly equal to the utility ratio, i.e., equal-cost increments of any two commodities must also be equal-utility increments.[11] The famous question of which determines price, util-

[11] It may seem like hairsplitting to stress the point, but all these equalities hold between ratios of commodity increments only. It is inaccurate to set up ratios between utilities or between costs, and the inaccuracy is dangerous, however conven-

ity, or cost, becomes then the question, which determines the other, the utility ratio or the cost ratio, or whether they are nearly or quite co-ordinate, as generally held and as, in particular, Professor Fisher holds for the interest rate.

It will not do to say without more ado that if two things are interconnected the control relation between them is mutual. If forces originating in or impinging upon *a* are always effective, *a* moving accordingly and carrying *b* with it, while forces arising in or impinging upon *b* are powerless to produce movement, *b* maintaining its relation to *a*, whether the latter is in motion or at rest, then it is surely justifiable to say that *a* determines *b* and not *b*, *a*. This may be due to mere difference in size. We should certainly say that the position of the dog's collar is determined by that of the dog, but that the position of the dog is determined by that of his kennel (or a vehicle) in which he is chained. The level of the ocean determines the level of the inlet and not the other way; the apple falls to the earth and not the earth to the apple (even though according to theoretical mechanics the total quan-

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ient stylistically. On the utility side, no psychological "magnitude" is really a quantity in the sense that one can be a numerical multiple of another. Consider temperatures, for example; only in terms of some theoretical physical measure can one thing be any number of times as hot as another. We do know the meaning of saying that one such magnitude is greater or less than another, but it is only at the point of equality that a truly quantitative comparison can be made. (Hence the "indifference curve" is the proper basic concept for all such reasoning.) The famous Weber-Fechner Law does not really contradict this view, and besides, psychologists do not apply it to valuations. On the cost side, if cost is taken as psychical cost, the same reasoning holds. If physical cost (means of production) is meant, cost is still unmeasurable, since the outlay is always a complex aggregate, never twice of the same composition, even as between different producers of the same good. Moreover, practically every element in what is called physical cost does involve psychological comparisons, different kinds of labor most conspicuously.

Another difficulty is the "marginality" concept—a very bad term for the relation between increment and aggregate. With regard to utility, two views are commonly held. The older and more naïve is that an increment of supply (which should always be a continuous stream and not a stock) makes its specific addition to the utility of the total, without affecting the utility of the earlier increments. This is the basis of the familiar utility curve with the implication of consumer's surplus. On the other hand, it may be held that the utility of all increments is always alike, the addition of each increment to the total bringing down the utility of the earlier ones to the level of its own. Both these views lead to nonsensical results: the first to

tity of motion is the same for both). In non-mechanical causality there need not be even a theoretical mutuality in the relationship. Can such a relation be shown for the case of commodity quantities of equal utility and cost, respectively?

The answer is that, in general, it can. In the general price problem the direction is the reverse for short-run and long-run views. In the short-run, changes in utility estimates are assumed to change consumption instantly and prices practically so. Under theoretical conditions, the real-cost and money-cost ratios immediately and continuously conform to the new price ratio. But this destroys the equality of yield of resources in competing fields of use (equality between money-cost and price) and makes it profitable to transfer resources. The transfer, however, proceeds rather slowly toward a position of equilibrium. During the shift the price and cost ratios must be said to be "determined" by the psychological comparisons acting on given rates of supply of commodities; but the position of equilibrium is another matter.

Since most of the productive services are not immediately

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fantastic magnitudes for total utilities and the second to the conclusion that the utility of a larger supply may be less than that of a smaller and consequently that people often choose and pay for a reduction in utility. The only defensible position seems to be to avoid total-utility implications altogether by sticking strictly to actual comparisons—between a certain increase in the supply of one commodity and a certain decrease in that of another.

On the cost side, difficulty may also be avoided by keeping clear of the notion of total cost and its changes. The cost which immediately affects the decision to produce, and hence alone affects price, is, realistically viewed, the increase in the total cost to some producer, involved in producing another unit. But if any producer has either increasing or decreasing real cost within his own establishment, the conditions requisite for theoretically perfect competition are violated, and it becomes impossible to make a general statement without specifying the precise cost curves of every single producer, or imposing the condition that they are all alike. For a general statement it is necessary to assume constant cost or have each increment of supply produced by a separate producer, so that changes in the cost of earlier units resulting from the production of later do not affect the decisions of producers of the later units—or else to drop competitive conditions and assume complete monopoly, when the discussion will run in terms of totals. When we speak of either utility or real cost as a quantity, as is sometimes stylistically unavoidable, they are assumed, both of them, to be measured in units of competing commodity. Thus the statement that commodities exchange "in proportion" to utility and to cost must be interpreted as meaning something definitely different from what it says.

transferable, those which are transferred are employed in very disadvantageous proportions with other resources in the expanding industry, and similar agencies left in the contracting industry are combined in very advantageous proportions. Thus the cost of the first product in terms of the second, as well as its price and money cost, is higher during the period of adjustment than it will be after the relatively fixed resources have time to become liquid and be transferred.

The causality is a matter of the relative responsiveness (amount of response in whatever time interval is under discussion) of consumers' behavior and producers' behavior, respectively, to changes in the conditions controlling their choices, the situation in which response is prompter and more accurate dominating the other. Consumers' judgments of utility affect their purchases quickly, and this is promptly reflected in changes in the prices, because the supplies change slowly, i.e., because producers' responses to price changes are slow. In the meantime, producers' costs reflect rather than influence the changed prices. Because price adjustments in the field of cost are sluggish and inaccurate and because bookkeeping standards are conservative, the effects on producers appear as "profits" in the expanding industry and "losses" in the contracting one. If adjustments were prompt and accurate they would appear as higher "costs" in the form of higher returns to (payments for) both mobile and immobile productive factors, in the expanding industry and lower costs, lower payments for the immobile factors in the contracting industry.[13] By similar reasoning, a change in production possibilities will but slowly make itself felt in changed supplies, then changed (incremental) utilities, then changed prices, simply because the actual readjustments in production require time, after the reason for

[13] The mobile factors would be paid at nearly the same rate in both industries. The costs which represent payments to immobile factors may be said to be "price-determined," or to come under the category of "surplus" or "rent," if the terms are understood in relation to the process of adjustment as described above. In the short run most of the productive services are in the immobile group. The distinction between mobile and immobile has little or no correspondence with any defensible division between "natural" and "artificial" productive goods and cuts sharply across "labor" as well.

making them arises. But if all responses and valuations are rational, the three ratios will be equal at every moment.

From the long-run standpoint, i.e., comparing one position of final stationary equilibrium with another (under given conditions as to wants, resources, and known technique), the case is totally different, because of a difference in the facts of relative change. There can be no question that, if sufficient time for adjustment is allowed, productive capacity can be transferred from nearly any use to nearly any other without serious diminution in its relative efficacy in the expanding industry. That is, the general rule for the long run under given general conditions ("static state") is approximately constant cost; the amount of one commodity which must be given up in order to produce an additional unit of another is generally not much affected by the relative amounts produced —given time for readjustments, and speaking of important industries and of changes within a range which does not take either of those affected out of that class. Under these conditions it is evident, first, that no matter what consumers' preferences may be or what may happen to them, production will bring prices and relative incremental utilities back to about where they were before, if technical conditions of production remain unchanged; and, second, that anything which happens on the production side, changing the relative productivity of resources in two uses (maintaining the law of constancy with varying output, at the new level), will ultimately establish this new relative productivity as the ratio of exchange and as the ratio of equal-utility quantities of the commodities. It is therefore correct to say that, in the long-run analysis, cost "determines" both price and utility.[13]

[13] This proposition, of course, should not be interpreted to mean that price as a quantity is equal to cost as a quantity or "demand" equal to "supply" in any of numerous meanings. Fundamentally, price, utility, and cost are, all three, ratios between quantities of commodities. These quantities are defined by the fact of being "equal" in three different relationships, and even this "equality" is not of a very clearly quantitative character. Logically, it would be better to avoid quantitative terminology. The condition of equilibrium in price relations in the long-run view (under "static" conditions) would then run as follows: The relative quantities of (any) two commodities which exchange against each other must be "the same as" the relative quantities which constitute indifferent amounts to every consumer of both, and also "the same as" the relative quantities which are alternative to each

III. THE DETERMINATION OF THE INTEREST RATE AS A PRICE

It should now be possible to make in brief compass the application of the general principles of price determination to the special case of the interest rate. The essential facts all hold good, and are even in peculiar degree accurate and indisputable. In particular, the crucial point as to relative flexibility or elasticity of consumers' and producers' choices between present goods and future goods is less open to question than in the case of ordinary present goods among themselves. The rate of interest is, to repeat, an exchange ratio between two classes or types of exchangeable entity, present and future, between services or goods rendering services "now" and services or goods rendering services at any future time or distributed in any way through future time, all types of futurity being arithmetically equivalent in a free market (p. 178, n.). The ultimate services themselves are assumed to be the same or equivalent to purchasers, and to differ only in the time of realization. (This view of wealth is accepted here "for the purpose of the argument.")

In all essential principles this price is determined in exactly the

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other in the choices presented to all productive services on a margin between them. (Always with reference to small increments, of course.) The "equality" defined by indifference to consumers is mainly one of psychical attractiveness. The one defined by the terms of producer's choices is affected by psychical factors but is in effect practically a physical fact, the relative yield of an increment of some transferable "resource" in the two fields. Allowance must theoretically be made (as Fisher fails to observe) for considerations like the "irksomeness of labor" (which more or less affect all factors) because of which the physical measure of the resource may not be final, so that it may take more or less than a day of use or performance in one activity to "equal" a day in another. (Under actual conditions, with the terms of choice and dispositions of men what they are, it is a familiar fact that the more "irksome" tasks are paid less per time or physical-energy unit, not more.) But any type of labor behaves much like an inert physical agency in its movements from one industry to another, falling into an occupation of the same "grade." The general principle must specify commodity quantities open to choice by producers on "equal" terms (or, again, "the same" terms). Producers change from one commodity (not productive agency) to another in production more slowly but over an indefinitely wider range than consumers do in consumption in response to the same price change. The long-run transferability of resources (roughly constant cost) gives the facts of producers' alternatives domination over the choices of consumers in fixing the ratios.

same way as any exchange ratio between two immediate goods, and it is a serious error of omission in a discussion of interest theory not to stress the fact. There are differences in detail in the conditions under which the principles work which require modification in the statement of the process of adjustment by which price is fixed, and these must of course be fully taken into account. But in an attempt to state the main essentials as briefly and clearly as possible, the bulk of the arguments which fill up the discussion of the interest problem can be passed over, to the decided advantage of the treatment. From Ricardo and Senior to Böhm-Bawerk, Fetter, and Fisher, this discussion has been cluttered up and the issue beclouded by theorizing (mostly quite bad) regarding the ultimate motivations involved in the choice between present and future ("spending" and "saving"—or "investing," which is not the same thing), on the one hand, and, on the other hand (not nearly so bad), regarding the technological nature and implications of the investing process. These have much the same relation to the essentials of interest-rate determination that a discussion of light and optics, on the one hand, and the physics of steam engine and electric dynamo, on the other, have to the pricing of electric-lighting service.[14]

[14] Most of the quarrels are over different words for the same thing—"abstinence," "waiting," "time-preference," "impatience," on the one hand; and, on the other, "productivity" of "capital" or of "indirect or roundabout process," etc. It has long been my own contention that the best form of statement to indicate the essential fact on the technical side is simply to say that time as such is a factor of production—the only really distinct, homogeneous "factor," as a matter of fact. Owing ultimately to the circumstance that there is a certain natural speed in physical changes, including movements of the human body and the functioning of nerves, sense organs, and brain, "preparations" which increase efficiency require time, as such, to make, and more time to exploit. The operation cannot be hurried, except at more than proportionally increased cost; outside of very narrow limits it is not possible to build a tool and wear it out on a reduced scale in proportionally reduced time, and within still narrower limits shortening the time means great loss in efficiency. And since it is another mere fact of the world in which we live that it is possible by making "preparations" to increase the ultimate efficiency of production, the same expenditure of productive capacity spread over a longer period of time can yield a larger result (and since it can, economic behavior does not admit of resorting to it unless it does). In other words, doubling the time over which given productive resources are used (doubling the amount of use) more than doubles the output.

Since we live in a material world, there is always some material carrier in which

The immediately pertinent facts regarding the interest rate as a price are, to repeat, identical with those affecting prices generally. The two types of goods exist. Purchasers distinguish between them, and they come into the market and are exchanged at a ratio which of course reflects purchasers' psychological preferences. But these in turn depend on supply, and both types of goods are freely producible, purchasers choosing between them, with some tendency to choose intelligently. In so far as they do so, they are guided in production by the value in prospect, and production is so distributed between the two uses that the greatest value obtainable in either field from a given outlay is kept

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the preparations are embodied and accumulated. Such things are "capital goods." During its construction, under competitive conditions, the value of any capital good is always equal both to the accumulated expenditure, including interest or what Professor Taylor well calls "carrying charge," and to the anticipated value of all its future services, discounted back to the moment of valuation. "Capital" is not a good term, as it may suggest either the total value of the capital good or that small fraction of it representing the peculiar element of carrying charge or payment for time (waiting). Moreover, human capacities form one of the chief modes of investment in the real sense; and (in a society prohibiting slavery) these are not produced under approximately competitive conditions, nor ordinarily and accurately capitalized in the market.

Time as a factor means actual, chronological, or elapsed time, and must not be confused with time as a dimension in the measurement of any productive service. Time or waiting is a cost exactly as labor is a cost, and it is nonsense to attribute the fact to psychological reluctance. Labor would be a cost, as long as it were productive, however much people enjoyed working. If they work less because of reluctance, the fact affects the price of labor and to some extent the prices of product into which labor enters in different degrees. But it is not certain that men could do much more work than they do if all irksomeness of labor were converted into pleasure. Similarly as to waiting. Productive conditions being what they are in the world, waiting would still be productive and hence still a cost of any particular commodity into which it entered, if people voluntarily postponed all consumption and saved their entire incomes. Capital like labor must be a cost and yield a return unless all its products are free goods. It is the necessity of choice between different uses which makes us economize anything.

The notion that value rests on pain is the great incubus in our inheritance from the theorists of the past—or from "common-sense"! It is not "difficulty" of attainment, but the necessity of giving up one thing to get another, which is at the basis of valuation. True, we give up leisure to get commodities; but the conditions of modern life leave so little freedom of choice in setting the margin that the fact has little to do with the supplies of commodities-in-general, and practically nothing to do with relative supplies of different commodities, which alone influences prices.

equal to that obtainable in the other. In view of the endless variety of specific opportunities in both fields, there can be no question that, given reasonable time for adjustment, transfers of productive activity can be made within any range actually in question without any important change in the "production equivalence" of the two types, the real cost of each in terms of the other, the amount of the one which is sacrificed in producing a given amount of the other. This relates to the two types of goods; there is a theoretically important qualification regarding the amount of income resulting from a given expenditure in producing income-bearing goods, in consequence of diminishing returns from capital; but the quantitative unimportance of this will be shown presently.

Assuming for the moment its unimportance, the conclusion is obvious. The interest rate is "determined" by this production equivalence; it is the ratio between an increment of net income per annum and the amount of present goods which offer themselves to producers as possible alternative results from the use of transferable productive capacity. In such a situation it makes no difference in the price what purchasers think of the relative desirability of the alternative products—as long as both continue to be produced; in this case it makes no difference in the interest rate what is the time-preference or impatience of any or all, so long as a net balance of investment continues to be made in the market as a whole. (Of course no price theory applies unless the goods actually come to market, and no cost theory unless they come through production at a cost.)[15] Under these conditions, which are those

[15] We are not here discussing interest theory with reference to another sort of world than this; but it may be observed in passing that the reasoning would call for little change to fit the reverse situation, a decadent instead of a progressive society.

The accumulation of capital goes forward not steadily but in waves, which are tied up with waves in the production and consumption of wealth in general. In any adequate treatment of price theory, much more than a footnote should of course be given to the phenomenon of waves, which has created such an uproar of late in the literature of economics as well as in the business world. In this sketch we can only remark that the importance of the waves for general price theory is often grossly exaggerated. The theory treats of the mechanism of an automatically governed economic system. There is no mystery in the fact that such a system shows oscillations in running; the best engine or heating system inevitably does that. Considering the conditions under which the governor of price competition works—necessity of adjustment to rapid growth, clumsy, lagging, and even inverse response of parts

properly described as "constant cost," purchasers' attitudes and acts "determine" the relative amounts produced and brought to market; they cannot and do not affect the price. This is the proposition which it seems needs to be put on the loud-speaker and screamed into the ears of economic theorists until they hear it.

All this in no way contradicts or invalidates the argument of the time-preference theorist (or the eclectic, like Fisher) that the interest rate measures the relative ("marginal") estimate of present and future on the part of every person in the market. That is perfectly true, but has little or nothing to do with "determining" the rate (infinitely little, assuming perfect adjustment, at a given moment). The consequence of any person's relative estimate of present and future is that he will buy or sell future income (with/for present goods) at the interest rate he finds in the market, up to the point at which his ("marginal") preference corresponds to the rate (i.e., until the increments which exchange for each other in the market are those which are "equal" to him). If the two commodities are forthcoming in practically unlimited quantities at any given relative real cost (accurately, relative increments having the same cost), the supplies will be such as to bring purchasers' preferences to the same equivalence, and that is the end of the matter as far as the determination of the price is concerned.

This relative cost is, however, itself a relative matter—the relative fixity of relative cost in comparison with relative utility. The question is whether an incipient change in price resulting from a change in consumers' choices will in fact freely change supply or will encounter resistance in the shape of costs, so that

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to changes in other parts, and the workings of a medium of communication manufactured in such a way as often to invert the proper responses—the mystery is rather that the waves are not larger than they are. The notion that the whole theory of automatic governing should be thrown overboard because waves appear with an amplitude of an appreciable fraction of the average speed is simply not to be taken seriously. This does not mean, of course, either that it was not a serious omission to fail to consider the waves in the general theory or that the detailed explanation of the waves as they manifest themselves (to say nothing of their control) is not a problem as difficult and important as anyone has contended.

(Much more than a parenthetical mention should be given to the specific effects of monetary changes in relation to "involuntary saving" and to speculation in long-term investments.)

the price ratio and utility ratio do a considerable or equal or predominant part of the moving involved in establishing a new stationary equilibrium. A definitive solution of the price problem involves a comparison of the two flexibilities. This of course cannot be made in terms of known measured data, but the character of the relations can be inferred from general considerations accurately enough that the differences in estimates will not affect the validity of the argument.

First, as to the psychological side, in the sense of the effects of greater or smaller purchases of future income upon its "marginal utility" in comparison with present goods. Is the supply of savings "elastic" in its response to changes in the price? As far as the writer knows, the contrary is universally admitted. No correspondence between rate of saving and interest rate has been shown, and the visible facts connect the amount of saving with many other considerations more intimately than with the return on investment. It is generally conceded that a considerable though unknown part of saving is probably inversely correlated with the return obtainable. It is surely safe to say that the elasticity of response is in the region of "small, if any."[16]

At this point, however, we encounter a confusion which vitiates

[16] Only a footnote can be given to the observation that in any case the psychology involved is not properly interpreted as "discount of the future" or "impatience." In a world in which pay is offered for waiting, people will naturally accept it for whatever waiting they decide to do at the price. And as long as waiting is productive, pay will be offered. No one can say what would happen in a totally different kind of world. It is a fact, however, that even with pay available, no inconsiderable amount of waiting, and saving and accumulation, is done without pay, even at a cost. There is certainly no ground for assuming any general principle of impatience as Fisher does (pp. 176–495). Neither the fact of payment for waiting nor the fact that people do not insist on postponing all consumption to the infinitely remote future proves that people are "impatient to spend." And as noted above, even if they did postpone all consumption indefinitely that would not mean the cessation of interest, if any motive for production be supposed to continue in force and production can be increased by preparations requiring time. Professor Fisher's "refutation" of the productivity theory in the version of Böhm-Bawerk overlooks the "detail" that the difference between having a satisfaction now and having it in the infinitely remote future is precisely the difference between having it and not having it. Even the difference between having it before and after one is dead is not very happily or realistically described as a mere difference in time. But the great bulk of actual accumulation depends on the latter choice; even the earnings of invested capital are

many treatments of the interest problem. The elasticity of saving is one thing and the elasticity of supply of "capital" another. (The "supply of capital" is of course the "demand for future income" and conversely—see below and n. 18). To have any picture of the supply, it is necessary to take into account the fact that the saving flowing into the capital market in any short interval of time is an addition to all the capital previously saved and invested in that market up to the beginning of the interval in question. The "elasticity of supply" must be measured with reference, not to variation in the amount saved in any time unit, in correspondence with variations in the interest rate, but with reference to the variation in the total amount to which the saving of that time unit is being added. There is no need to go into detail as to the method of computing that total in order to see that the variation in it is an infinitesimal of the second order in comparison with the variation in the increment. If the latter variation is questionable as to sign and certainly small, the variation in the total is negligible, *a fortiori*. The elasticity of supply of capital must be taken as practically zero; at any given moment, or within any short time-period, the supply is an almost completely fixed quantity.

On the production side—the cost of future income in terms of present goods, which is the demand side of the price situation when "capital" is the commodity priced—the question of elasticity cannot be disposed of quite so briefly. It will presumably be admitted that productive power may be freely transferred within a moderate period of time from the making of present goods to the making of capital goods, within any limits practically significant, without encountering any significant increase in the cost of the

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in large part not "enjoyed" in the only sense allowed by Professor Fisher's theory, but reinvested indefinitely.

As previously stated, we do not know whether people generally, or the class from which savings come, would "discount" the future or the present or neither, "other things being equal." In general, there is perhaps more ground for the inverse allegation as against the modern European peoples and especially the Teutonic stock (and the West European Jews?), namely, that they "look before and after, and sigh for what is not" and neglect the present moment. Compare Faust and Rabbi ben Ezra with Marius the Epicurean, or the Puritan with the Greek view of life.

latter in terms of the former.[17] But the cost curve for capital goods is not precisely the issue. As more productive power is supposed to be diverted to that field, it is evident that, other things being equal, a somewhat different type of goods as such will be produced with each successive instalment, and that the whole process of investment is subject to "diminishing returns." That is, each successive equal increment of present wealth given up will theoretically yield a somewhat smaller increment of perpetual income. This means increasing cost of income in terms of present goods, which means reduced elasticity of demand for "capital," since capital is the amount invested and is measured by present goods. To establish the position here maintained it is necessary to show that this effect is negligible, measured on the scale of the magnitude involved, as a base; that is, that the increase in the cost of future income being negligible, the potential "demand for capital" is practically infinite.

To avoid confusion in thinking through these somewhat intricate relationships, it is necessary to keep in mind that the thing priced is the "capital" and the price (the interest rate) the perpetual income which it is expected to yield (expressed as a percentage of the capital). The capital itself (for the purpose of this problem) is what is bought with the "money" which is concretely the subject of the loan, namely, productive capacity to be used in constructing capital goods ("goods" in a very broad sense, of course). This productive capacity is measured by the present goods it is capable of producing, and may be regarded as a quantity of present goods sacrificed, though no previously accumulated stock of present goods is involved, as implied in some of the "classical" literature. Because in the interest problem the thing priced is simply wealth expressed in money, instead of a concrete thing being priced in money, it results that the "supply" and "demand" sides of the situation exchange places as compared with the ordinary price problem. The supply side is the psycho-

[17] Space limits preclude discussion of the possibility that the curve might actually be one of decreasing cost. But the fact that the comparison is between large and heterogeneous classes of things, all the items in both classes being for the most part "freely reproducible," assures constant cost.

logical side and the demand side the technological, or production, or objective side of the situation. If the market quotation priced future (perpetual) income in terms of present dollars the relation would be the usual one.[18]

In order to discuss intelligibly "elasticity of demand for capital" as thus defined, namely, the relation between the amount which the market could absorb (under given conditions) and the price, it is necessary to have some more definite way of taking

[18] I should like to call especial attention to this inversion of supply and demand because I have been confused with regard to it in previous utterances and find a similar confusion in discussions by other writers.

The above definition of capital should be brought into relation with the earlier statement (p. 198, n.) that the essence of capital is time. Time is merely the "peculiar element" in "capital" as it results from investment, i.e., in the capital good as a value magnitude. This is always greater than the amount of present goods "invested," because the investment itself takes time, but the excess is a small fraction of the total. The real conversion is not, strictly speaking, that of a sum of present wealth into a stream of future wealth, but of a large stream for a short period into a perpetual stream.

It might be an improvement in the terminology for dealing with these somewhat elusive concepts if we distinguished between "capital" before and after investment, or between capital-for-investment and actual productive wealth. One way of viewing the entire interest problem is to ask what determines the amount of capital in capital goods. The essence of the productivity theory is that under continuous free and intelligent investment this is "determined" by their cost of production. Of course it more immediately reflects capitalization, but the position of equilibrium is set by technical or objective conditions in production, just as in the case of any two commodities produced under constant cost. Fisher rightly emphasizes that interest and rent are not different shares in distribution, but the same share viewed under different forms. But as ordinarily quoted in the market, the two are very different. Rent includes provision for perpetual maintenance, which for short-lived capital goods is several times the interest; the latter begins only after all amortization elements are deducted from imputed earnings. In addition there is a speculative factor; interest is a guaranteed money payment and differs from rent-less-depreciation by a margin of "profit" or loss.

The assumption of perfect competition also involves perfectly free flow of new capital into investment at the rate measuring potential opportunity. In consequence of various sorts of friction, and especially the business cycle, the theoretical momentary rate of interest cannot be identified realistically. The argument from theory receives general confirmation from the fact that cyclical oscillations affect chiefly the short-period contract, the yield of long-time loans being enormously more stable. (Perhaps it "ought" to be more affected than it is by prospective changes in the price-level; but the meaning of rational behavior under conditions of uncertainty is to me an unsolved problem.)

account of the total accumulation. But first it is imperative to be clear as to the place of this magnitude in the setting of the price problem. The one important difference between the price analysis in the case of interest and that of ordinary prices arises from the fact that saving and investment is a cumulative process. It is a phase of economic growth, outside the framework of the conventional "static" economic system, unfortunately so called. The production of an income-yielding investment is essentially like the production of an indestructible good. An investment does not in fact yield an income unless it provides for perpetual maintenance and replacement, with an excess, and only this excess is properly income at all. Thus all income from produced wealth is perpetual income, and the production of income-yielding wealth means intrinsically an addition to the total supply. The replacement of such wealth, whether with new units of the same kind or of a different kind, is not properly production. The case is categorically different from that of bread or shoes, which flow into the market only to flow out at the same rate into consumption, which is to say annihilation.[19]

Yet it is possible, legitimate, and necessary to consider the character and meaning of "static" demand and supply curves for capital, if one is to explain the interest rate as it is at any time, i.e., at any point in the course of the growth in the capital supply. The demand curve is that made familiar by J. B. Clark in his exposition of economic statics, and assumed in every exposition of productivity theory by the method of hypothetical dosing. In the accompanying sketch it is drawn in a way to bring out its historical character. Some point N on the curve must be taken to represent "now" or whatever date in history is under discussion. The

[19] There are, of course "consumption goods" of the same character, such as art works, gems, and the like. The precious metals are partly consumed and partly accumulated. If space permitted it would be interesting to compare the principles of the interest rate with those of the "value of money." The result would be in general of an opposite character. Gold is subject to "increasing cost" and to such violent fluctuations on the use-value or psychological side that its value must be said to "determine" its (marginal) cost of production at any time. But, again, if we stick to the view of goods priced in terms of gold, the analogy with the interest rate is parallel and not inverse.

portion of the curve above and to the left of this point is fixed in, to indicate its uncertainty in a quantitative sense. It might be thought of in at least three different views, the historical investment and accumulation, or what would happen if the capital of a

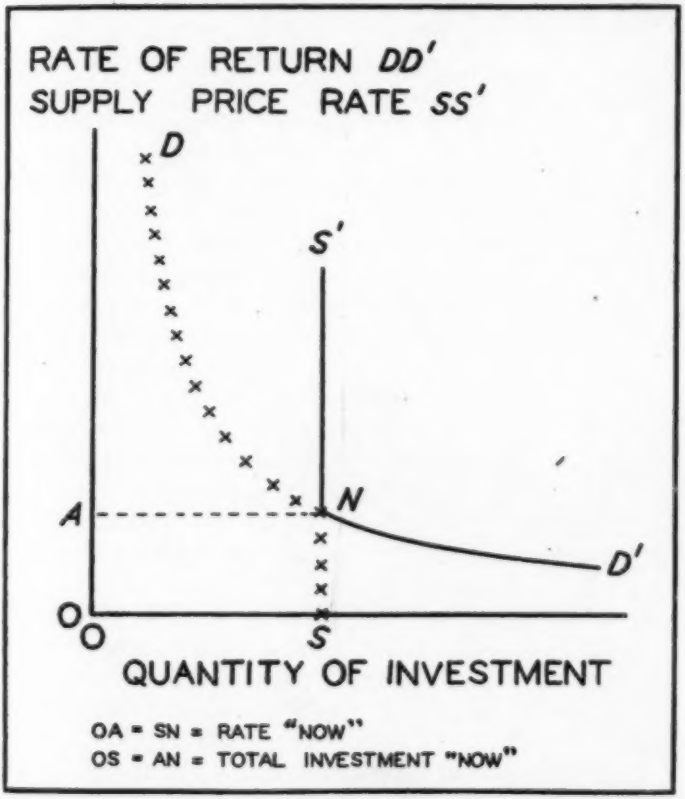

FIG. 1

society could be removed, leaving the other factors intact, and "dosed" back onto them, or what would happen if the capital were withdrawn in the realistic sense in successive increments. Not only are these views theoretically different, but none of them can be made the basis of a curve showing quantitative facts; and

yet the differences among them, or among estimates relating to them, will not affect the present argument. The only essential point is that the total supply of capital is "very large" in comparison with possible variations in it, and that the opportunity for further investment is on a similar scale with the total.

This demand curve DD' is first to be thought of as drawn on the assumption of *caeteris paribus*, the timeless or static curve. It is fundamental that actually variations in the capital supply in the direction of increase are alone relevant. The significant demand curve is the potential demand for further investment, represented by the solid portion of the curve or ND'. This curve will theoretically have some downward slope, showing diminishing returns from investment under given conditions.[19] The precise shape of the theoretical, momentary supply curve is problematical, but it is certain that no variation in the amount of supply potentially forthcoming at a moment, corresponding to a hypothetical change in the interest rate, would be visible on a diagram drawn to scale. Hence the curve is a straight line parallel to the axis of price (SS'). (It might, if the facts were accurately known, incline infinitesimally to the right, but again it might incline infinitesimally to the left.) It must be understood that as any real "production of capital" actually takes place, this supply curve moves bodily to the right, and its invisible deviations from the straight vertical may change in character. The effect of this movement on the location of the intersection point, indicating the interest rate, must be discussed under two heads: (*a*) what would happen if the demand curve remained unchanged, and (*b*) what does happen, or has happened in the past and may be considered probable for the future.

Under the first head the essential point is presumably obvious if the situation itself is grasped. No conceivable change in the capital supply in the static sense can affect the rate to an extent remotely approaching visible dimensions. But to be at all realistic, we need to consider the possibilities of growth over a moderate interval of time rather than the theoretical potentialities at a

[19] If the diagram is drawn to scale and the curve is one of constant elasticity, it does not matter whether N is taken at one point or another along the curve. It is placed rather low down and to the right to make the argument visible to the eye—the only point in using the diagram at all.

moment. To do this it is needful to take some figures for the total investment up to "now" and for the rate of growth. It is all merely for illustration, and accuracy is even more unimportant than it is unattainable. Let us consider the United States as a capital market (ignoring importation and exportation of capital) and take the investment hitherto at the estimated national wealth, say four hundred billion dollars.[21] Then, let us take as a round number and a conservative one, an "elasticity of demand" equal to unity. (The Douglas-Cobb formula indicates a value of $1\frac{1}{4}$, and this is not improbable as an estimate.) On these suppositions it would require a further investment of four hundred billion dollars to bring the interest rate down to half its actual level (whatever that is). One-twentieth of this amount, or twenty billions, is certainly a high estimate of the actual saving in the country in a year. Manifestly no possible variation in the amount saved in a year could make enough of a variation in the total supply to affect the interest rate visibly.

And a year is a considerable interval to use instead of a point of time, already a considerable step toward the more realistic problem of the actual trend. It is to be emphasized that if we really adhere to the ("static") premises of *caeteris paribus* the result of the saving falling to zero would be merely that the interest rate would remain absolutely stationary, while if the saving increased to any amount it would be merely a somewhat more rapid fall. If the entire income produced were saved, the fall should be some five times as fast and it would still require infinite time to bring the rate down to zero. The conclusion from reasoning which assumes a fixed demand curve is that capital accumulation leads to a slow fall in the interest rate, that savers' choices can at most exert a very little effect on the rate of the decline. The cumulative result would be detectable after a considerable number of years.

Turning briefly to the real course of events, and its reasonable implications, the fact which strikes every student is that the large additions to the capital supply characteristic of modern times do not actually bring the interest rate down at all, as a demonstrable

[21] Space precludes discussion of the possible rôle of "gifts of nature" in this total. In my own judgment, the figure is definitely too low for either historical cost or replacement cost of our national wealth, if estimates were made on a defensible business basis, i.e., too low for the produced wealth or capital.

generalization. What happens is that social progress in its other phases than capital accumulation moves the demand curve for capital upward as the supply curve moves to the right, so that there is no clear vertical drift of the intersection point. The two sets of changes tend to balance each other, sometimes one predominating for a time and sometimes the other. Whether this is an accident, or whether some "law" of psychology or history underlies it, such that new supply of capital "creates" new demand (or possibly new demand calls forth new supply), cannot be discussed here. But it must be noted that in the light of the evidence it is inadmissible to argue, as Marshall does, for a movement toward an ultimate equilibrium where the rate will cease to call forth new saving. (Presumably Marshall is here, as elsewhere, unduly influenced by the earlier classical writers.) One must be cautious in the use of the assumption of "other things being equal." It is not permissible to take as given other things which are known to be causally dependent upon the things assumed to vary. In the present case, the tendency of progressive accumulation to bring down the incentive to save is necessarily opposed by its tendency to reduce the resistance to saving, since the savings invested yield new income (to owners of other factors if not to the savers).

Other assumptions relating to the interest rate as an incentive are questionable, though not demonstrably false. Finally, it seems reasonable to believe that there is a vague but real causal connection between falling interest rates and "invention" in the broad sense of experimentation leading to improvements in technical and business methods. Such reflections cannot here be followed out beyond suggesting that long-run, historical changes must be faced as problems of historical causality and treated in terms of concepts very different from those of given supply and demand functions and a tendency toward equilibrium under given conditions.[22]

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[22] The unfortunate use of the term "economic statics" has been referred to more than once. It ought to apply to the conditions of equilibrium in a market, the first stage of the usual mathematical development of price theory. And the proper mean-

APPENDIX ON THE ARITHMETIC OF INTEREST

A clear view of the essentials of the interest phenomenon is obtained by considering the computation of interest in the simplified situation of an individual economy, sticking to the view that the rate is the quotient of the net annual yield of an investment over the investment itself when both are maintained constant in perpetuity. Suppose that Crusoe builds a tool or makes some improvement, which costs 25 days of labor, increases his productive efficiency 5 per cent, and lasts 5 years. Take 300 work days for a year and call the investment 25 "units" (days' labor or days' product, amounting to the same thing). If no outlay for maintenance or repairs is necessary, the imputed product per year will be 5 per cent of 300, or 15 units. The simplest way to hold investment and return constant is to set aside out of each year's product one-fifth of the investment, or 5 units, as a "sinking fund" to replace the implement at the end of its service life. This leaves a net yield of 10 units, which is 40 per cent of the investment, and that is the rate of interest, assuming that the sinking fund is unproductive. If 5 days must be spent each year in keeping the agent in repair (and if it is unproductive during that time), the imputed product of 15 must be reduced by a maintenance cost of 105 per cent of 5, or 5.25 units, leaving 9.75, which after deducting the sinking-fund instalment leaves for interest 4.75 units, or 19 per cent. Taking l to represent a year's labor (or a year's product without the tool), e for the per cent increase in daily product due to use of the tool, c for the construction cost, m for annual maintenance cost (man and machine time, all measured in l), and n for the service life in years, then the rate of interest is $i = \frac{el - m - c/n}{c}$.

Since it is hardly reasonable to suppose that an idle sinking fund would be accumulated in this way, it is a more realistic view of the problem to assume

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ing of "dynamics" in economic theory is the study of movement under given conditions of some sort, whether or not it tends toward equilibrium. Its problem would be the measurement of forces and resistances and velocities of change. Problems involving changes in the fundamental conditions of economic life correspond to nothing in mechanics, which assumes permanently fixed constants—as well as composition of effects which is also inadmissible in evolutionary or historical economics, as it should be called, instead of economic dynamics.

The "last word" of this overlong yet overpacked paper should repeat that its aim has been less to straighten out interest theory than to assist in extinguishing both the doctrine of pain cost and that of utility as determinants of price in general. Assuming economic freedom and intelligent behavior, two principles of value are "axiomatic": (*a*) things equivalent in use must be equal in value, and (*b*) things to be had on equivalent terms from the environment must be equal in value. The user is judge of the equivalence in one case and the producer in the other. Consumer and producer choices bring the two equivalences to equality. The question which determines the other (and the price) is that of the relative freedom of consumers to shift from one commodity to another without changing equivalence in use, in com-

that the sinking fund can be invested at the same rate. The computation on the same principle of keeping investment and net yield constant will now necessitate replacing the c/n in the above equation by an expression for the annual contribution d to sinking fund necessary to accumulate the original investment at the end of the life of the implement. Assuming interest at the rate i compounded annually, this expression is, of course, $d=\frac{ci}{(1+i)^n-1}$. Making the substitution, the entire expression readily reduces to the more usual amortization formula showing equality between capital and present worth of income instalments, r per year for n years, $r\frac{1-1(1+i)^{-n}}{i}$.

Using the value of r from our example, namely 9.75, solving by the method of successive approximation yields an interest rate of 27.26 per cent.

The essential point is, of course, that nothing representing or suggesting time-valuation ever appears among the data in the computation. As shown in the text, the only chance for play of this motive is that people may invest more or less at the rate afforded by the opportunities of the situation. Over a long period, under imaginary conditions, the character of the choice made "might" slowly lower the rate. But what actually happens to the interest rate depends on nearly everything else that happens in the course of history.

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parison with the freedom of producers to shift from one to the other without changing equivalence in "cost." In fact, the freedom is much greater in the second case (in the long run) and to a corresponding degree "cost determines price," but it is almost purely a technical cost, measured in units of the other commodity itself. It is essential to grasp that these principles hold for indirect goods as well as direct and are unaffected by complementarity in use (in "consumption" or "production") so long as proportions are variable, which is a practically unimportant reservation. Hence they apply to the prices of productive services, or the distribution problem, as well as to ordinary prices.

It is worthy of note that the relative flexibilities of consumption and production have in large part a basis of historical social psychology or "institutions." At any given time people care relatively much what they consume, and relatively little what they produce with their labor or property. To a trifling extent in a civilized country this is a matter of physiological need; it is more a matter of aesthetics, but most of all one of habit or fashion. Of course physical specialization of productive capacities, human and non-human, must be allowed for. But the egregious error of viewing cost in its relation to price as a matter of pain, effort, or sacrifice, instead of the relevant facts of mobility, has led to enormous exaggeration of this element, as well as misplacement of its true rôle. The latter is much more important in connection with human services than in connection with "natural agents."

THE GERMAN-POLISH RYE AGREEMENT[1]

DIRECT governmental participation in international syndicates for the purpose of stabilizing prices, such as the steel, coal, and nitrate cartels, has been hitherto rare. But the programs recently adopted by certain European countries to protect themselves against the agricultural depression tend to promote public as well as private international pools, especially on the part of those states which have established some form of grain monopoly. The first of the interstate syndicates of this character is the German-Polish Rye Commission, which was set up by the Rye Agreement of February 18, 1930.

The immediate cause of the Rye Agreement was the desire to reconcile the conflicting interests of two rival grain-exporting nations. Agrarian reforms in both Germany and Poland had led to governmental policies for the artificial stabilization of home markets. Both governments were pledged to drastic paternalistic legislation. It was thus but another step in the development of these policies to attempt the completion of the process of stabilizing home markets by pooling German and Polish sales in a demoralized export market where these countries found themselves in sharp competition. At the same time the Agreement was drawn across a grave political background. On the one side is found such obstacles to international co-operation as the German reluctance to accept the eastern frontiers as a permanent settlement, the constant agitation for a revision of the Treaty of Versailles, and the severe import restrictions against all Eastern farm products. On the other side there is the nationalistic aggressiveness of the

[1] This article is largely based upon interviews and data recently obtained by the author in Berlin from the German Minister of Food and Agriculture, the Reichskommissar in der Reichsforschungsstelle für Landwirtschaftliches Marktwesen, the German and Polish members of the German-Polish Rye Commission, and various grain-traders, editors, and bankers; and in Warsaw, from the Polish Minister of Foreign Affairs, the Minister of Agriculture, the president of the Polish Landowners' Association, and various officials and traders. The office of the United States Department of Agriculture in Berlin and the American financial adviser in Warsaw also gave generous assistance.

newly created state of Poland, which eagerly desires to prove its right to exist and which unfortunately, under the Paris arrangements of 1919, has been saddled with the military guardianship of Eastern Europe, compelled to watch Germany on the front and to guard against Soviet Russia in the rear. Ever since 1925 there has existed a desperate tariff conflict between the two countries, with destructive consequences to trade; and although a flimsy treaty to end this chaos was signed on April 17, 1930, there seems little likelihood of its immediate ratification. In view of this situation, the existence and continuance of the Rye Agreement appears all the more remarkable.

Political prejudices often give way before economic necessity. In the present case both countries have been equally faced with the urgent need to stabilize their domestic rye markets by an orderly elimination of their respective surpluses. The fortunes of the rye-growers in Germany and Poland are closely related. The principal rye districts of the Reich lie east of the Elbe toward the Polish border, while the best grain districts of Poland are along the German frontier. Weather conditions in both regions are parallel; wet seasons and dry seasons are equally shared. After bad harvests neither region has a large surplus to export, while good seasons bring bounteous crops, portions of which must seek markets abroad. The seasons of 1928 and 1929 were particularly good. Large stocks of rye accumulated, while the home price steadily declined. In the foreign market, Polish rye in competition with German rye was quoted at a price below what the Polish Landowners' Association claimed to be the average cost of production.

The disturbing factor in the rye situation is the fact that after the World War the markets—both domestic and foreign—have become narrowed while the rye acreage in Germany and Poland has been little curtailed. Following the republican revolution of 1918, the German people have eaten more wheat bread than rye bread. Similar changes occurred in the foreign markets. Before the war both Germany and Russia exported considerable quantities of rye to Scandinavia, Denmark, and Holland; but during the war, when German and Russian exports declined, Scandinavian and Danish consumers became accustomed to wheat as a substi-

tute for rye and are now disinclined to abandon the more luxurious grain. The use of denatured rye (eosin-rye) for stock-feeding has been pushed by the German government, and this market has somewhat increased, but not sufficiently to relieve the rye-growing landowner. In the meanwhile, campaigns to induce the Prussian farmer to restrict his rye acreage and increase his wheat acreage have not produced immediate benefits.

For the past five years grain exports have been artificially stimulated by the use of *Einfuhrscheine*, or import-certificates. Since the *Einfuhrscheinsysteme* has played such an important rôle in the agrarian program of Germany, an explanation of the theory and practice of this politico-economic device will throw light on the situation which produced the Rye Agreement of 1930. The history of import-certificates goes back to the period of the German industrial revolution in the latter half of the nineteenth century, when the empire changed from a grain-exporting into a grain-importing country.[2] Decreases in the cost of transportation had brought American, Russian, and Indian grain fields into competition with German farms even in the home market. Protective measures were demanded; and under Bismarck, in 1885, a high tariff was levied on wheat and rye. But this attempt to benefit German agriculture was not an unmitigated success. It is true that the tariff raised the price of grain in Germany, but the increase was not commensurate with the protective rates. In the western and southern provinces prices rose to the level of the tariff wall. But in eastern Germany the price-increase averaged less than one-half of the new rates.

The reason for this divergence was found in the fact that the cost of transporting grain from eastern Germany by water into the English and Scandinavian markets was less than the transportation of grain by rail into the western and southern provinces. Moreover, the eastern grain of Germany had a smaller gluten con-

[2] For the history and operation of the *Einfuhrscheinsysteme* see Friedrich Beckmann, *Einfuhrscheinsysteme*, "Volkswirtschaftliche Abhandlungen der Badischen Hochschulen" (Karlsruhe, 1911), N.S., No. 1; Siegfried Tschierschky, *Neuordnung des zollfreien Veredelungsverkehrs* (Jena, 1904); and articles on "Einfuhrscheine" and "Identitätsnachweis" in *Handwörterbuch der Staatswissenschaften* (4th ed.; Jena, 1923), I, 353–55; V, 377–82.

tent and bread-making value than foreign grain imported along the Rhine through Holland. The price of grain in the western and southern provinces thus equaled the world-market price plus the tariff; while the price in eastern Germany equaled the price in the western and southern provinces minus the cost of overland transportation across the Reich. As a matter of fact, every increase of the tariff wall saw less foreign wheat and rye imported through Holland, and also less German grain exported through Koenigsberg, Danzig, and Stettin. But overland transportation charges absorbed most of the new profits of the eastern grain-growers.

The import-certificate system was an ingenious scheme to correct the above-mentioned difference in price development between eastern and western Germany. By the law of 1894, import-certificates were issued to the exporters of wheat, rye, barley, oats, and legumes, and these could be applied to the payment of tariff duties on the same kind of grain imported anywhere in the Empire within six months. By later acts, import-certificates were made valid for the importation even of other grains than those for which they were issued; and, of course, they were negotiable instruments and were bought and sold on the open market. By this device, the eastern German grain surplus was encouraged to seek the Scandinavian and English markets by way of the Baltic ports, while foreign grain, paying tribute to the German exchequer, flowed into western and southern Germany. The statistics are significant. German wheat exports, which had been 6,547,000 bushels in 1880, had fallen in 1893 under the high tariff system to 11,000 bushels. But in 1894, the exports of wheat rose to 2,910,000 and increased to 10,842,000 bushels in 1900. Rye, oats, and barley showed parallel developments. At the same time, after 1894, the importation of all grain was immediately doubled. Thus, the *Einfuhrscheinsysteme* brought Germany again into contact with the world-market in spite of the existing high tariff.

The result of all this was to lessen the difference between the price of grain in the Koenigsberg and Cologne markets and to raise the average price throughout the Empire, redounding apparently to the prosperity of the farmer and the middleman, if not to the milling industry. The World War imposed only a suspen-

sion of the import-certificate system. Under the Agricultural Law of August, 1925, the system was revived, and the validity of the certificates was even extended from six to nine months.[3] The following years witnessed the expected increase in both the exportation and importation of grain. In the meanwhile, the price of rye on the foreign markets continued to fall and prompted the German government to seek artificial means of international stabilization. The German "plan economy" had already begun to move toward a grain monopoly. In 1925, a governmental office was organized under the name of the German Grain Trading Company (Deutsche Getreide Handelsgesellschaft), which now divided the export trade with the individual traders and co-operative concerns. Uncontrolled exportation was shortly to give way to a policy of governmental concentration.

In the autumn of 1929, the Polish government adopted a policy similar to the German *Einfuhrscheinsysteme* and attempted to enhance the domestic price through stimulus to exports by means of an export bounty. To rye-exporters were issued export-premiums worth about 10 zloty (5 marks) for every 100 kilograms of outgoing rye. The stimulated export trade was not, however, handed over to free competition; an export syndicate, known as the Polish Government Grain Office, was established for the administration of the premium system. This expensive device of using the taxpayer's money to promote rye exports for the maintenance of the home price was defended on the ground of the need for saving an important section of the nation from utter ruin. And, of course, it has always been expected that the premium system would not remain a permanent burden on the national budget. There is no doubt that the system advanced rye exports and tended to stabilize the home market. But it contributed to the demoralization of the foreign market and introduced cutthroat competition with German rye. With the exception of a short interval, the price of rye in the Scandinavian, Danish, and Dutch markets steadily declined throughout the year 1929.

At the end of the year 1929, the Deutsche Getreide Handelsgesellschaft and the Polish Government Grain Office held large

[3] *Reichsgesetzblatt*, Jahrgang 1925, Part I, No. 44, p. 331 (September 12, 1925).

stocks of rye, the production of rye in both countries was on the increase, and each faced a ruinous campaign of competition in the northern markets. At this juncture negotiations were opened between the two syndicates looking toward a common administration of the German-Polish rye export with the aim of avoiding price cuts in the foreign markets. In January, 1930, the two governments themselves entered the negotiations directly, and a provisional agreement for three weeks was made whereby the German government purchased 20,000 tons of Polish rye for delivery in the foreign markets while Poland agreed to issue no export premiums pending the conclusion of a permanent agreement. The German Ministry of Food and Agriculture seriously contemplated the acquisition of the total Polish surplus with a view to reducing competition with German rye on the Scandinavian markets. The terms offered, however, were not acceptable to Poland; at the same time the policy was severely criticized in Germany on the ground that it assured Polish exportation at the expense of German agriculture and that it failed to take into account the possibility of Russian dumping which was likely to prove a more serious obstacle than Polish competition.[4] Accordingly, the negotiations took a new turn. An international pool, including Germany and Poland, was proposed. The new program would admittedly be subject to the depressing effect of Russian dumping, but at least it would avoid squandering the national fortunes in a German-Polish grain war and might be able by means of a concentration of offers to secure more stable prices. Negotiations on this basis led to the signing of the German-Polish Rye Agreement effected by an exchange of notes at Warsaw on February 18, 1930.

The new agreement was to extend from February 10 to July 1, 1930, with the expectation, of course, of further extensions if the experiment proved successful. By a second agreement signed on July 7, this extension was actually advanced to July 1, 1931. By these two agreements, a Deutsch-Polnische Roggen-Commission (Polsko-Niemiecka Komisja Zytnia) was established, consisting

[4] Cf. *Berliner Börsen-Courier* (evening edition), December 18, 1929, p. 9, and February 20, 1930, p. 10; *Deutsche Getreide-Zeitung* (Berlin), December 14, 1929, p. 1.

of three German and three Polish delegates, later enlarged to include four delegates from each country.[5] Each government undertakes to give the Commission practically a monopoly of rye-exportation by agreeing to grant import-certificates or export-premiums only for such quantities of rye as are sold through the instrumentality of the Commission. The Agreement does not prohibit private exports of rye, but renders such exports practically impossible inasmuch as the syndicate rye is given the exclusive right to receive German import-certificates and Polish export-premiums, the certificates representing a value of 9 marks per 100 kilograms and the premiums 10 zloty, or 5 marks.

As a matter of practice, price-fixing by the Commission has followed a simple and automatic procedure. Two members, one from each side of the Commission, meet daily and examine bids received from their foreign agents. The bids are accepted in order of their amount and in the designated proportion as between Germany and Poland. Hence the price fixed by the Commission for "Commission rye" is, after all, the foreign market price. Each government is at liberty to choose the way in which to bring its grain trade into the control of the Commission. For this purpose the business activities of the Commission are carried on through a limited liability company.[6] The German partners of this company are the German Grain Trade Company (Deutsche Getreide Handelsgesellschaft) acting as trustees for the German interests, the Polish partners being the Polish Association of Grain Exporters (Vereinigung der Getreide-Exporteure der Republik

[5] The headquarters of the Rye Commission are in Berlin in offices at Tauben-Strasse 25. The delegates, acting under instructions from their respective governments, meet daily. On the side of Germany, the members include: Dr. Fritz Baade, Reichskommissar für Landwirtschaftliches Marktwesen, representing the Reich and serving as president of the German members; Herr Hirsch, representing the Getreide-Industrie & Commissions A. G.; Herr Prager, representing the wholesalers; and Herr Rossa, representing the co-operative societies and the new corn monopoly. The Polish delegates do not sit as representatives of any groups but represent the state without distinction of affiliations. Herr Rosciczewski, formerly a ministerial councilor in the Polish Ministry of Agriculture, serves as president of the Polish delegation. The remainder of the delegation include Herren Radoniski, Goldman, and Rosenberg.

[6] Berlin, Trade Register Department B, No. 43,873, founded on April 7, 1930.

Polen) acting as trustees for the Polish interests. The two governments are at liberty to appoint other partners to act as trustees for the interests of their countries and to subdivide the interest. Any modification of the charter of the limited liability company or its statutes is subject to the approval of both governments. The expenses of the Commission are borne out of a commission-charge of one-tenth of a mark levied upon every 200 kilograms of rye sold through the Commission.

The quantities of rye exported through the intermediary of the Commission are distributed over the exportation trade of both countries, giving Germany 60 per cent of the total and Poland 40 per cent. The procedure of distribution is as follows: (1) The sales of rye, which during the months of July until January inclusive do not exceed 8,000 tons per day, are to be distributed in equal parts between the two countries; from February to June inclusive the equal quantity is reduced to 6,000 tons daily. (2) The sales of rye of more than 8,000 tons daily (reduced to 6,000 tons in the period of February to June) are to be distributed respectively: 70 per cent for Germany and 30 per cent for Poland until the total ratio of 60 per cent for Germany and 40 per cent for Poland is reached. (3) The quota for each country is recorded at the end of each business day and is balanced at the end of each second calendar month; the quota not fully taken up by either country at the end of each two months' period is not to be usable in the following two months. (4) The back-claims on the quota are to be credited in such way that 80 per cent of the sales of each day are to be distributed between the two states as provided in Sections 1 and 2, while the remaining 20 per cent of the sales of each day are allotted to the country which has not made use of its quota in the past six months and thus is in arrears.

The Commission has complete authority to fix the minimum prices for the exportation of rye according to the market situation. This minimum price refers only to rye daily traded in under the name of "German-Polish rye." For other quantities of rye for either immediate or future delivery the minimum price is fixed by the Commission in comparison with the price of "German-Polish rye." On request of the representatives of each of the two

countries the minimum prices can be increased by a maximum of 5 per cent if in the course of the preceding ten days a certain total for the two countries is sold, this total being more than 40,000 tons during the months of July to January inclusive, and more than 30,000 tons during the months of February to June. In similar manner the minimum price can be reduced by a maximum of 5 per cent if the past ten days show a total sale of less than 35,000 tons during July–January, and of less than 25,000 tons during February–June. Finally, the Commission is compelled to sell rye as far as possible only to dealers and consumers in the country of destination (c.i.f. sales). If, occasionally, sales f.o.b. or sales in transit of the German-Polish frontier are made, such sales must be only to such firms as pledge themselves to keep the Commission constantly informed as to the resale of this rye within the countries of destination.

The German-Polish syndicate is largely limited to trade through the Baltic. Excluded from the jurisdiction of the Commission is German rye sold by the land route to Czechoslovakia, Austria, Switzerland, and France. Polish rye may be sold without reference to the Commission over the land frontiers of Latvia, Lithuania, and Esthonia, and over the Polish southern frontiers as far west as Pavonkow. But these shipments may not be transshipped to the Oder River or carried to the Baltic or North Sea harbors. Small frontier traffic is also exempted from the jurisdiction of the Commission. The Commission is under obligation to investigate any charges that rye has been exported in violation of the Agreement; and if it is found that the Agreement has been infringed in one country, the government of the other country has the right to terminate the arrangement after a fortnight's notice.

The distinctly voluntary character of the Agreement is emphasized in the further arrangements for termination. For instance, the Agreement can be annulled by either party upon a fortnight's notice in case import-certificates or export-premiums are abolished in one of the countries, or in case the internal price in either country falls so low that traders undertake exportation without the aid of import-certificates or export-premiums.

The voluntary nature of the Agreement is also emphasized by

the fact that in price-fixing there must be unanimity. Each government has equal representation on the Commission; there has been no provision made for an umpire; and unless both countries are agreed, no price can be established. But the Commission, meeting in daily conference, has not found it difficult to perform its functions. Tables I and II indicate the minimum prices fixed by the Commission from March to August, 1930, and the amounts of rye sold under the two agreements.

The Rye Agreement was adopted, as already related, with the expectation that exportation would be facilitated and the home market stabilized in both Germany and Poland. Expectations were realized in Poland, but not in Germany. Unprecedentedly large American and Canadian stocks depressed the world-price, while Russian rye, thrown on the market regardless of economic conditions, demoralized the trade. For instance, late in May, the Soviet government suddenly dumped 60,000 tons of rye on the Rotterdam market at a figure 8 marks below the prevailing price. It required weeks for the trade to absorb the influx; and in order to prevent this cheap rye from invading Germany the Reich Minister of Food and Agriculture immediately raised the tariff from 90 marks per ton to 150 marks.[7] At the same time the price of rye in the Scandinavian markets continued to decline. When the Agreement was signed, rye brought 6 gulden c.i.f. in Scandinavian ports; by May the price had fallen to 4.80 gulden. At this figure rye was sold below what German and Polish experts claimed was the average production costs in both countries. The German Minister of Food and Agriculture took the position that, no matter how the Poles felt about it, Germany could not throw away her national fortune in this prodigal fashion. Accordingly, by the ordinance of May 6, 1930, the Minister of Finance was empowered to discontinue the issuance of free import-certificates for rye exports in case the total exportation exceeded the amount of 70,000 tons before the new crop came on the market.[8] By July 5, the

[7] *Reichsgesetzblatt*, Jahrgang 1930, Part I, No. 18, p. 175. This ordinance of May 22, 1930, was signed, of course, by both the Minister of Finance and the Minister of Food and Agriculture.

[8] *Reichsgesetzblatt*, Jahrgang 1930, Part I, No. 15, p. 163.

TABLE I

MINIMUM PRICES FIXED BY RYE COMMISSION IN 1930

Date	Denmark	Latvia	Netherlands Belgium	Drawski Nlyn	Danzig	Posen	Sweden	Finland	France
The Crop of 1929									
March 1–10	Hfl. 6.00	R. 100.0	Hfl. 5.75	Zl. 16.00	Zl. 16.00	.....	.....	.....	
March 11–13	5.00	85.0	5.00	15.50	Hfl. 5.15	.....	.....	.....	
March 14–24	5.20	98.0	5.90	16.90	.....	.....	.....	.....	
March 25—April 5	6.40	106.0	6.50	.....	.....	.....	.....	.....	
April 6–20	6.25	.....	6.00	.....	Hfl. 5.90	Zl. 17.75	Hfl. 6.90	.....	
April 21–30	6.10	105.0	6.10	.....	.....	16.00	.....	.....	
May 1–16	5.25	89.0	5.20	13.15	.....	15.00	.....	Hfl. 5.35	
May 17—June 6	5.16	80.5	4.65	13.15	Zl. 14.20	12.75	5.40	.....	
June 7–27	4.70	79.0	4.55	12.20	Hfl. 4.00	12.75	4.80	4.85	
June 28—July 10	5.00	84.0	4.75	13.75	Zl. 21.25	.....	.....	.....	
July 11–18	5.60	96.0	5.70	14.50	Hfl. 4.75	.....	.....	.....	
July 19–31	6.10	100.0	6.00	17.00	Hfl. 5.60	.....	.....	.....	Hfl. 6.50
August 1–8	5.85	97.0	5.70	16.70	Zl. 17.25	17.00	.....	.....	
August 9–15	6.15	100.0	5.85	16.70	.....	16.50	6.00	.....	
The Crop of 1930									
July 21–31	6.40	.....	5.85	16.25	.....	16.00	.....	.....	
August 1–8	5.95	97.0	5.90	17.00	Zl. 18.25	17.00	.....	6.20	
August 9–15	6.10	102.0	5.90	16.75	Zl. 17.25	16.25	5.95	.....	6.20

certificates were exhausted and German exportations came to an end. The Poles, as entitled under the Agreement, continued their exportations.

It should be remarked that another consideration entered into the motive for the decree of May 6, namely, a restriction of funds available for the maintenance of the *Einfuhrscheinsysteme*. For the fiscal year April 1, 1930, to March 31, 1931, the Ministry of

TABLE II

SALES OF RYE UNDER THE RYE COMMISSION IN 1930 (TONS)

German Rye Harvest of 1929		*Polish Rye Harvest of 1929*	
March	11,879	March	28,355
April	13,481	April	11,955
May	30,725	May	16,590
June	41,599	June	22,510
July	15,385	July	29,490
August	.....	August	6,445
Total	113,069	Total	115,345
German Rye Harvest of 1930		*Polish Rye Harvest of 1930*	
July	.....	July	4,080
August	14,745	August	56,865
September	725	September	41,800
October	.....	October	29,855
November	.....	November	4,155
December	.....	December	8,810
Total	15,470	Total	145,565
Total German exports	128,539	Total Polish exports	260,910

Food and Agriculture was able to secure no greater allowance from the national treasury than 102,000,000 marks.[9] The value of the free import-certificates issued to exporters of rye was fixed at 9 marks per 100 kilograms, and the total amount of these certificates was to be a charge against the allowance. At the same time, free import-certificates were issued to exporters of barley, oats, and live stock, and thus the requirements for these obligations compelled a restriction of certificates issued on rye exports.

Late in September, however, the government reversed its agricultural program and suspended the *Einfuhrscheinsysteme*, with

[9] Compare article v of the *Gesetz über Zolländerungen* of April 15, 1930. *Reichsgesetzblatt*, Jahrgang 1930, Part I, No. 12, p. 133.

the immediate effect of cutting off a large part of the exportation of all agricultural products. The new policy—a part of Chancellor Bruening's program of retrenchment in all departments—is dictated by the view that under present conditions the continuation of the payments on exports entails an unsupportable burden on the federal treasury. Whether the new policy is temporary or permanent, and whether it will lead to the termination of the Rye Agreement, remains to be seen.

The Rye Agreement has indeed been considered with some justification as the logical conclusion of the German agrarian system supported by the Social-Democratic party and championed in the writings of Dr. Fritz Baade. This policy has produced: the German Trading Company, with its extensive operations in the purchase of grain; the Corn Monopoly, created for the purpose of promoting the purchase of domestic rye for stock-feeding in preference to imported maize; the new Bread Law of July 17, 1930, which requires the mixture of 10 per cent rye in flour and which will increase the consumption of rye in Germany by nearly a million tons per year; and the governmental effort to force raisers of live stock to feed eosin-rye (rye treated with a harmless rose-colored chemical which renders it unfit for human food) in the place of imported barley and maize.[10] In the latter case, in a renewed effort to force eosin-rye upon stock-feeders, the government has recently halved the import duty on barley to such consumers as agree to use an equal quantity of domestic eosin-rye and potatoes for stock-feeding purposes.

The objections to the Rye Agreement which abound in Germany are the same strictures directed at the state monopoly of grain.[11] It is held that only those products can be syndicalized the manufacture of which can be subject to a permanent control.

[10] Concerning the Bread Law of July 17, 1930, see a statement by Dr. Schiele, Reich Minister for Food and Agriculture, in the *Deutsche Getreide-Zeitung* (Berlin), August 20, 1930, p. 1.

[11] See articles by Dr. Justus Schloss, Dr. Alfred Stern, and Dr. Otfried Mielck, in the *Deutsche Getreide-Zeitung* (Berlin), February 20 and 23, April 5, May 7, June 12, 14, 18 and 21, July 18 and 24, August 19, 21, 24 and 28, September 28, October 23 and 29, 1930. Cf. articles by Dr. Paul Bondy in the *Berliner Börsen-Courier* (evening edition), February 18, 1930, p. 9, and February 20, 1930, p. 10.

Certainly grain cannot be counted among such products, for the farmers' choice of acreage, as well as the weather, rain, frost, and drought, are not subject to governmental dictation. Hence, an international syndicate, as well as a national monopoly, is bound to fail. It is also said that, if the Agreement should for a time succeed in raising the price of rye in the foreign markets, the production of rye will be stimulated, not only in Germany and Poland, but also in all rye-growing countries. Even the attempt to raise German and Polish prices to a higher level abroad will result in competition from Russia, Rumania, Hungary, Czechoslovakia, Canada, the United States, and Argentina. Attracted by the export prices effected by the German-Polish syndicate, these countries will be encouraged to increase their acreage and to invade the German-Polish markets. Finally, it is contended by the independent grain-traders that the "red tape" of an international commission prevents the exporter and the importer from acting promptly no matter how kindly the commission may be disposed; its decisions cannot be foreseen, and it has no live contact with international business. In particular, the complaint is made that the Getreide-Industrie & Commissions, A. G., or organized grain-dealers, by their representation on the Commission, secure all the offers to the detriment of the independent traders.

In Poland, the Agreement has met with enthusiastic approval. Under its régime Polish exports have made a marked advance. In 1926–29, the three years previous to the Agreement, Germany exported 906,000 tons of rye and Poland only 164,270 tons; the proportion being 84½ per cent for Germany and 15½ per cent for Poland. But under five months of the Agreement, Poland exported 154,900 tons in comparison to Germany's 118,269 tons. It is true that most of Polish rye was sold at a price which the Germans refused to take. Nevertheless, Polish rye moved into foreign ports, and the home market was proportionately benefited. There has been no increase in the home price of rye; but on the other hand, the slump in price has been apparently checked.[13] Even more important, the sharp decline which has hitherto immediately followed the new harvest was avoided this summer. Govern-

[13] Cf. *Vierteljahrshefte der Polnischen Landwirtschaft*, I, No. 4 (July, 1930), 175.

ment loans to farms may partially explain the escape from the usual decline, but Polish opinion widely attributes it to the operation of the Rye Agreement.[13]

The recent German suspension of the *Einfuhrscheinsysteme* has not resulted in the termination of the Rye Agreement. The new policy aims at effecting a more close adjustment between production and demand. It is designed to render Germany more nearly self-sustaining as a grain-producing nation. Energetic campaigns are now under way to reduce the rye surplus by a decrease of the rye acreage and to bring up the wheat production to the domestic consumption by an increase of wheat-planting. At the same time the German people are impelled by law and propaganda to eat more rye bread and less wheat bread; while special inducements are held out to stock-feeders to buy rye instead of barley and maize. The ultimate success of the new policy will mean practically cessation of the importation of wheat and diminution of the exportation of rye. Under such a policy the usefulness of the Rye Agreement is reduced to a minimum. But the vagaries of climate and agricultural conditions will probably result in occasional surpluses which must either be stored or exported, and in the latter event the Commission will serve a useful purpose. Likewise, any lessening of the devastating competition of Russian rye in the narrow Scandinavian markets—due to a breakdown of the collection and transportation systems or due to poor crops—would invite the resumption of German exports. Since last July the Poles alone have carried on against Russian competition. During the first six months of 1930, when the Commission served both the Germans and the Poles, it tended by means of concentration of offers to promote a stabilization of foreign markets. And in any case it has proved a convenient device for international co-operation. Through its agency the Germans and the Poles have refrained from cutting each other's throats in the Baltic competition and are prepared to co-operate at any time in the future.

In conclusion a few words should be added regarding the relative position of the German-Polish Rye Agreement in the realm

[13] Statement made to the author by Prince Lubomirski, president of the Landowners' Association in Warsaw.

of world politics. There is some expectation that the national grain monopolies which have appeared in the last few years will lead to international economic undertakings of far-reaching effect. Complete grain monopolies now exist in Norway, Latvia, and Esthonia. A partial monopoly is found in Switzerland. Germany, France, Italy, Spain, and Czechoslovakia have systems of grain regulation which are intimately controlled by the state. The bounty-system has been adopted in Poland, Hungary, Jugoslavia, and Rumania. In Soviet Russia, the largest grain-producing country in the world, the government has a complete monopoly of foreign trade; and under its ambitious Five Year Plan it expects to play an increasingly important part in the grain trade of the world. The United States has set up the Federal Farm Board with vast powers of regulation, while a large part of Canadian and Australian wheat is marketed through a pool. The tendency of the past several years had been to bring grain into a world-economy administered in a large degree by governments. This situation opens directly to international combinations through governmental negotiation. The several conferences of the eastern European states at Warsaw, Sinaia, Bucharest, and Belgrade in the autumn of 1930 look toward an international grain cartel. Significant are the negotiations under the aegis of the League of Nations in respect to grain tariffs and agrarian finance, and likewise significant are the committees on wheat and agricultural credits set up by the conference of twenty-seven European states which met at Geneva in January, 1931, to consider Briand's proposal for a Federal European Union. It cannot be predicted that these movements will lead to permanent or even temporary systems of co-operation in grain-marketing or, in case the systems actually appear and develop, what form they will take. But in any event the experience of the Germans and the Poles in their effort to secure a limited international concentration of grain surpluses will offer a practical lesson in agrarian statesmanship.

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SOME THEORETICAL ASPECTS OF STOCK-MARKET SPECULATION[1]

A BROADER understanding of the processes involved in stock-market fluctuations has developed during recent years. The nature of the interactions within such fluctuations and their economic significance have been widely discussed in the financial press. Many points are clear even in the minds of those who depend upon the financial section of the daily newspaper for their economic education.

The state of this knowledge, however, is still far from satisfactory. Progress has been confined largely to details; the problem as a whole is not yet solved. The present study is intended to be a modest contribution to the economic theory of fluctuations in the stock market—a theory which, indeed, requires further development. The study has been based upon general principles which may be said to be rather widely accepted in modern economic thought. Since these principles are held to apply sufficiently and correctly to the facts of economic life, the theory outlined in the following pages should not conflict with the facts of the particular field studied, unless there be a flaw in its logic. It would seem natural, therefore, to test our theory and to show that it sufficiently describes, and thus explains, the empirically determined movements and magnitudes involved in stock-market fluctuations. However, the writer believes that we lack enough long-time data to permit adequate testing of such a theory statistically. Until we are provided with such data, then, it seems best—in the interests of theory and statistics—to avoid empirical proofs and counterproofs of such problematic and erroneous nature as have been applied, for example, in the case of the quantity theory of money.

The problem before us is of highly practical interest. The stock-

[1] Credit is due Carl Theodore Schmidt, of the University of California, not only for the translation of this article but also for valuable constructive criticism of the rough draft of the manuscript.

market crash of October and November, 1929, at London and New York, together with repercussions in Continental European exchanges, has again aroused the interest of economists and business men. The question which confronts us may be formulated somewhat as follows: What are the economic effects of exaggerated stock-market speculation and of the profits and losses resulting from extensive booms and sharp recessions?

"Economic effects," for purposes of this discussion, may be defined as reactions upon the supply of economic goods.

The details of our question are as follows: (*a*) What does the appearance of differential profits and losses[2] mean in the realm of economic goods? (*b*) What effects, if any, do booms and crashes have upon the supply of capital? (*c*) In what way, if at all, do booms and crashes influence the distribution of incomes and, therefore, the production of goods? To these questions, which will be treated most easily under the fiction of a self-sufficing economy,[3] we shall add: (*d*) What is the economic significance of such profits and losses, assuming that international economic relations exist?

Most readers of trade journals are aware today that differential profits and losses on the stock market are "paper" profits and losses, that such profits and losses do not in themselves mean the creation or destruction of economic values, and that "what one gains, the other loses." Now, such a view tends to overlook or to minimize the economic importance of stock-market fluctuations. This importance, however, is related to capital supply, distribution and production, and international relationships—that is, it is concerned with our last three questions. So long as we are dealing with our first question, then, we must recognize the force of that argument. Differential gains and losses of this kind have no parallel reflection in the world of goods; they do not mean, *ipso facto*, production or destruction of goods. Millions "gained" or "lost" on the stock market do not imply equivalent gains or losses for the economy. All this is quite evident. But it would

[2] By this phrase we shall always mean "differential profits and losses resulting from extensive booms and sharp recessions."

[3] It is obvious that no essential point will be neglected by this simplification of the problem.

be a serious mistake to assume that such gains and losses have no consequences of economic significance.

What effects do booms and crashes have upon the supply of capital? Economists have often discussed the question whether or not the stock market ties up capital. No doubt, many pertinent answers have been given. However, the argument especially stressed several years ago, and most rigorously expounded by Professor Cassel, is quite fallacious and serves merely to obscure the small grain of truth which doubtless is contained within the popular conception. Cassel's answer to the question is well known. The stock market does not tie up capital at all, he says, for each buyer turns over funds to a seller, who again invests such funds. In the final analysis, all of the capital put into the stock market flows on into the "real capital market"—through a chain of buyers and sellers the last of whom places the funds at the disposal of the market for "real capital." The error involved in this view will be indicated in the course of the study, without direct critical examination of Professor Cassel's viewpoint.[4]

In reality, the situation is as follows: The turnover of shares on the stock market ties up means of exchange in exactly the same way as does the turnover of goods on the commodity market. It would never enter the heads of economists to assert that the turnover of goods does not tie up money simply because each buyer is confronted with a seller who passes on the funds realized, ultimately back to the source of their origin (let us say, a central bank). Money in the broadest sense (i.e., means of payment) is technically needed to facilitate the exchange of values. This technical function of circulation, of course, ties up money.[5] Now an increased gross value of the turnover of shares on the stock mar-

[4] Many economists realize that Cassel's argument is fallacious. Thus, for instance, B. M. Anderson says that "the contention that money loaned to the stock-market speculator comes back to industry in one way or another is at least equally true of a commercial loan or of a real estate mortgage loan." (*Chase Economic Bulletin*, May, 8, 1929, pp. 14 ff.)

[5] "Means of payment," "means of circulation," "money"—these are used as synonymous expressions. "Money," according to the theoretical viewpoint of the writer, is used in the broadest sense possible—the term relates to every acceptable medium of exchange, and its true character is that of a claim to economic goods. Metal money, bank notes, government notes, credit instruments of every kind, are embraced within this concept of money.

ket implies, *ipso facto*, an increased circulation of money, since only such an increase can allow the rise. Thus stock-market transactions tie up means of payment in exactly the same way as do transactions in the commodity markets.

Capital, according to Schumpeter's definition, is "that sum of money and other means of payment which is available for the purposes of entrepreneurs at any given time."[6] The means of payment (for instance, bank credit) that are tied up by the turnover of stocks during a boom period—*in addition* to those means ordinarily required—are doubtless not capital in this sense. "They do not appear in the aspect implied by the term 'Capital' "[7]—that is, they do not serve entrepreneurs as the means of drawing productive goods away from traditional uses into new productive purposes. They are, therefore, technical means for the turnover of economic goods. However, the direction of funds into the stock market, for the fulfilment of that technical function, reduces the sum "which is available for the purposes of entrepreneurs at any given time." Thus, although the stock market does not strictly tie up "capital," it does have "effects upon the supply of capital."

Now, as we said, the turnover of stocks does tie up means of circulation, and an increasing gross value involved in such turnover means increasing circulation of money (always in the broadest sense) on the stock market. This increase of the flow of money into the stock market—within the elastic, but nevertheless real, limits to the creation of additional purchasing power—reduces the reserve of credit available to entrepreneurs for new productive enterprise. And, since we term these funds "capital," our answer to the second question is that *the release of loan funds into a speculative stock market restricts the supply of capital available for productive purposes.*[8]

[6] ". . . . jene Summe von Geld und anderen Zahlungsmitteln, welche zur Überlassung an Unternehmer in jedem Zeitpunkte verfügbar ist" (Joseph Schumpeter: *Theorie der wirtschaftlichen Entwicklung* [2d ed., 1926], p. 173).

[7] ". . . . sie erscheinen nicht unter dem Aspekte, den das Wort Kapital *verkörpert.*" (*ibid.*, p. 172).

[8] It is true that credits granted for stock-market speculation usually remain within the banking system, unless they are consumed as profits of sellers of stocks. But they cannot be loaned on, over and over again, to their full extent. Once grant-

A declining stock market at best implies a replenishment of the "capital reserve" to the extent of a previous reduction, and ordinarily only to a smaller extent. Part of the loan funds extended during the preceding boom is usually spent for consumptive purposes by sellers who have profited, and therefore cannot be refunded out of the stock market.[9]

Before we enter into a discussion of our third question, we must say something regarding the really important function of the stock market which is emphasized by theorists who oppose artificial restrictions of stock-market activity. It cannot be denied that a large share of capital is handed the entrepreneur through the stock market; that, to a large extent, the capital market is opened to productive enterprise via the stock market. The practical mechanism that, in the form of stocks, provides the lender with a short-term asset and the borrower with a long-term credit, is, in fact, a very useful and indispensable instrument. But all this is beyond the consideration of our study. So far as the funds directed into the stock market serve to take up *new* issues, so far as they simply *pass through* the stock market on their way into production, they cannot cause extensive boom periods with increasing turnover at rising prices. It is only this latter phenomenon which concerns us here.

We now turn to the third aspect of our problem: What are the effects of excited stock-market speculation, of boom and crash,

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ed, they re-appear as *creditors* against which a certain reserve must be kept and whose corresponding assets, therefore, cannot be loaned on altogether, but only after deduction of the reserve ratio. In other words, the extension of credits reduces the liquidity, and consequently the lending power of the banks even if all of these credits remain within the banking system. For, they remain there, *not as repayments of the respective debtors, but as deposits of other creditors* against which a certain, though not full, reserve has to be kept. The "fund of purchasing power" is thus narrowed, not to the full extent of the credits granted, yet to some considerable extent. And also quantitatively this is important.

[9] Of course, it *can* be refunded out of income from sources other than the stock market. The successful speculator, the "winner," is in a position to pay back his credit at the time when he spends part of his gains. But the "final loser" cannot do this, and it is he who remains debtor to the "capital fund." It should be noted that the spending of part of the speculative profits would in itself lead to a stock-market decline because of the outflow of funds were it not for the additional inflow of new credits. This is an outstanding phenomenon of boom periods.

upon the distribution of incomes, and therefore upon consumer demand and such production as may be governed thereby?

The reduction of the supply of capital by a boom on the stock market also affects production in that it prevents, at least in part, new productive enterprise that otherwise might have been undertaken. It thus affects the production of things which are essentially new and for which there is as yet no "demand experience," either in quantity or quality. Now, in our third question, we have reference to such production as is governed by established demand. This distinction is important. Both kinds of production lead to the supply of goods. The latter tends to maintain existing standards of the supply of goods or to adjust them to changes in demand; the former works in the sense of discontinuous, abrupt economic progress.

We ordinarily observe in the case of extensive stock-market speculation that, on the one hand, differential profits are partly spent in consumption, and that, on the other hand, differential losses are partly covered from incomes otherwise spent in consumption. It is this which interests us in dealing with the third question.

The flow of income originating in price differences on the stock market is partly spent in consumption and thus directed to the commodity markets, where it appears as additional purchasing power. This, in itself, should lead to increases in the general price level, because, as we have said, those differential incomes have no parallel in the world of goods. The expenditure of funds created through credits for new enterprises also tends to raise the general price level. But in this case a productive contribution to the flow of goods is made subsequently—a contribution that tends to bring the price level down again. While the effects of the new purchasing power upon general prices are temporary in the case of productive utilization, they are final in the case of the spent differential incomes. Thus such incomes, when spent or to the extent spent in consumption, through their effects upon the price level reduce the "real income" of the recipients of incomes from other sources. Merely the claims to goods are increased in number, while the complementary flow of goods remains un-

changed. *The distribution of the flow of goods* (real income) *is thereby altered*, and this has important consequences. For the expenditure of those differential gains by no means is directed to all individual classes of goods in proportion to their share in the total national production. Preference is given to certain goods or classes of goods which ordinarily are consumed only on the higher levels of individual income. Under the pressure of this demand, production must shift from goods hitherto relatively preferred to goods *now* preferred. No doubt, such transitions are reflected in social consequences. However, those differential incomes are neither equable nor permanent. Not only do they not last, but their place is taken eventually by differential losses. These, also, relate to the third question of our study only in so far as they are refunded out of incomes otherwise spent in consumption. Although differential profits ordinarily result in an increase of consumption more quickly than differential losses result in a restriction of consumption, the latter phenomenon yet is a fact and of sufficient real importance.[10] There is again no parallel in the world of goods when the purchasing power, otherwise consumed, is drawn away from the market of goods and disappears. The price level tends to decline; the "real income" of the recipients of incomes from other sources tends to rise. A transition in production again reflects the pressure of new price formation, since the markets for certain goods become less active.

This transition of production is of great economic importance. In compliance with the changes in demand a shift in the direction of production must take place: new plants, new means and methods of production, become necessary, old ones superfluous; labor is newly employed here, set free elsewhere; fields of long experience are abandoned, and in others the producer has to feel his way toward a new equilibrium. Such a transition involves losses of efficiency and of time, and increases of frictions. *The forced adjustment of production to situations which, in principle, are tem-*

[10] Since the expenditure of differential profits is anticipated, in boom periods, by production, it is clear that the cessation of such profits alone will affect production. It is sufficient merely to mention this; its consequences are similar to those of differential losses.

porary in nature and therefore subject to readjustments leads to economic sacrifices. Exaggerated stock-market speculation in the sense of our third question, therefore, results in economic loss.

Such is the answer to the third question. It is incomplete because this study is interested in the effects upon economy as a whole. Some significant aspects, therefore, must be ignored. Thus, for example, we have ignored the effects of stock-market speculation upon the supply of individual groups of income recipients with goods. But these effects are also of importance, even directly for economy as a whole. For instance, the possibility to make money by speculation may draw away forces from productive occupations. Here it should be mentioned that it is by no means unimportant what goods constitute a nation's production. Nowadays an economy may be said to be better supplied the larger the number of more primitive wants satisfied, rather than wants for luxury goods. That is, we must consider an economy favorable when the individual satisfiable wants do not diverge too greatly from one another. This divergence is called forth by the discrepancies between individual incomes; differential incomes increase essentially these discrepancies and establish wants, the satisfaction of which draws away forces from the production of more primitive goods. And if, in addition, these differential incomes are achieved through no productive contribution at all, it is obvious that not only the social, but also the economic, critic finds fault with them. Curtailment of "real income" of producers in favor of non-producers is bound to be injurious to social production. This point is worth detailed consideration but cannot be developed in this article.

Now, before turning to a final résumé, we shall consider the economic effects of exaggerated stock-market speculation on international interrelations. We therefore leave the methodological fiction of a self-sufficient economy. All effects already discussed remain the same when international relations exist. They will not therefore be mentioned again. At first glance it would seem that our reply to the first question—What do differential profits and losses mean in the world of goods?—would not be modified by the new factor of international interrelations. There is no parallel in

the world of goods to arithmetical profits or losses, no matter whether they relate to foreign or domestic speculation. In the strict sense of the question, this is doubtless true. However, the differential profits that accrue to foreigners signify the acquisition of a claim to a part of the domestic flow of goods, the realization of which does not simply change the distribution of real income, but draws away goods. And, on the other hand, that which domestic speculators gain from foreigners increases the domestic fund of goods. Differential profits and losses of foreigners, therefore, involve direct, real economic losses and profits for the *national* economy. Differential gains from foreigners are a gain of goods, differential losses to foreigners a loss of goods.

This ends our short analysis. Its conclusions may be summarized as follows:

1. Exaggerated stock-market speculation has unfavorable effects upon economy. Differential profits and losses, in a self-sufficient economy, *ipso facto*, do not signify an increase or decrease in goods. However, in the case of foreign speculators, such profits and losses do involve real losses and gains of goods.

2. Extension of credit to a highly speculative stock market reduces the supply of capital available for productive purposes, while a "crash" at best releases funds to the extent of what was previously borrowed (and cannot increase the supply of capital above the original amount).

3. Differential profits and losses, to the extent that they are spent in consumption or refunded from incomes otherwise consumed, force readjustments of production which result in economic sacrifices.

We have considered the more important economic aspects and effects of highly speculative stock-market movements, but not all the effects. These effects, under the qualifications offered, are cogent, mechanical effects; they not only can, but are bound to, occur whenever those suppositions are realized. However, there are also effects which occur *ordinarily*, although not *necessarily*. These effects are psychological in nature and as such often overshadow the effects analyzed in this study. The "boom" atmosphere stimulates the spirit of enterprise and increases the possi-

bilities for new stock issues. The panic atmosphere of the crash lames many enterprising forces. These effects are well known; and it is, therefore, superfluous to describe them in detail. Their net resultant, too, is disadvantageous to economy. The optimism of boom periods will hardly facilitate desirable enterprise; it will hardly expand the limits of productive credit to an extent that such advantage would not be more than offset by the effects of a panic-crash which frequently even endangers the financial basis of old and sound enterprises.

All this, I believe, explains why economy has more to lose than to gain by long periods of exaggerated stock-market speculation. The theoretic basis of a banking policy opposed to such activity is not so weak as business frequently, and science at times, believe. The eloquence of facts, I am sure, speaks in its behalf. To prove this by statistical evidence, as soon as our equipment with data permits, will be one of the most important tasks for future studies in this field.

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OTHMAR SPANN'S SOCIAL THEORIES

OTHMAR SPANN relates in his *Kategorienlehre* how in his youth he was asked to make a statistical investigation of illegitimate children. In connection with the task he was struck by the idea that illegitimacy may be looked upon as a social phenomenon in either of two ways:

(1) genetically, according to its origin, or (2) functionally, according to its social consequences in society. Genetically, illegitimacy was seen to arise out of physiological and psychological conditions; and thus, in so far as one regarded the conditions causally, illegitimacy might be studied as a phenomenon definable by its cause; but, with respect to its significance in the organic totality of the social life, illegitimacy showed itself in an entirely different light.[1]

Careful analysis of the situation showed clearly that it is the accomplishment, function, rôle, or performance which determines the social characteristics of illegitimacy and the nature of illegitimacy itself as a social phenomenon. This contrast between the genetic and the functional viewpoint holds for every social and economic phenomenon—for example, particularly for value and price. The genetic view contributes nothing to sociological understanding, which comes rather from the functional conception exclusively. This same distinction between causal and organic methodology which Spann first applied in his economic investigations is also necessary in the study of society.[2] Most modern schools of sociology find the essence of society in the interaction of the individual members, one might say the social atoms.[3]

[1] Othmar Spann, *Kategorienlehre* (Jena, 1924), pp. 6 ff.

[2] Spann developed these arguments for the first time in his "Zur Logik der socialwissenschaftlichen Begriffbildung" in the *Festgaben für Friedrich Jul. Neumann*, (Tübingen, 1905). Later and better-known works also deal with the problem; see *Haupttheorien der Volkwirtschaftslehre* (20th ed.; Jena, 1930), which appeared in the United States under the title, *History of Economic Thought* (New York, 1929); *Fundament der Volkwirtschaftlehre* (4th ed.; Leipzig, 1930); *Gesellschaftslehre* (3rd ed., Leipzig, 1930); and *Gesellschaftsphilosophie* (München and Berlin, 1928).

[3] For the conception of relation, see Simmel, *Sociologie*, pp. 5 ff., and A. Vierkandt, *Gesellschaftlehre*, p. 28.

This idea of interaction, however, deprives sociology of its proper subject matter by attributing reality only to the members and endeavoring to conceive these in terms of causality. The collective unity or totality of society is in this view a mere abstraction, a mere derivative, instead of something real on its own account. And, if there is no unitary whole, the idea of function or performance also loses its validity because it is the whole which constitutes (*darstellt*) the realm of purpose toward which functions are directed.

Thus Spann was led to make his well-known fundamental distinction between the individualistic and the universalistic views of society. The essence of the distinction is, in brief, that according to the individualistic doctrine the parts of society—that is, the individuals—are prior to the whole, whereas according to the universalistic conception the whole is prior to the parts. Of course this does not mean temporal priority but logical priority as expressed in the saying of Aristotle, "For the whole necessarily precedes the part."[4] The whole is prior—that is, more important—for the reason that a part could not exist without the whole. The hand, according to the example of Aristotle, is a hand only so long as it performs its function in the living body; on the dead body it is no longer a hand but only a mass of flesh and bone. In the same way when a man is outside the economic order, he no longer performs the economic activities of a member and is merely a biological or psychological entity. On the other hand, as a member of an economic order he is not a biological or psychological entity but a member of this whole. We shall try to show by a more detailed treatment of the individualistic and universalistic principles how this relation influences the theory of society.

The main tenet of individualism is that the individual alone is the primary and original reality in society; that the real substance of society is found in the individual and not in the organization of individuals into a higher unity; that the only reason for the existence of society is to be found in individuals; that consequently society consists entirely of individuals; and that over and above the individual no independent social reality is to be found. The

[4] *Aristoteles Politik*, I, 1, §11, translated by Susemihl.

clearest example of this theory is found in the philosophy of Hobbes: to explain state and society he postulates a primitive or natural condition. In this stateless condition men live as solitary beings, each sufficient to himself in his own autonomous existence. Each is in a state of war against all of the others (*bellum omnium contra omnes*). To end this unbearable condition the individuals band themselves together in a state; in other words, they form the state by entering into a social contract. This example shows clearly how individualism thinks of the individual, namely, as a spiritually self-sufficient entity which develops and forms itself out of itself alone and only on external grounds determines upon the formation of a state. Whether or not the state developed historically in this manner is, of course, of no importance. Under this conception of the individual, the nature of society is clear; it is a mere juxtaposition of individuals, a heap, sum, conglomerate, or whatever term seems most descriptive. Thus society becomes a thing without sense or purpose; for, if the individual is spiritually complete and self-contained, society is unessential, society is merely something external, instrumental, utilitarian. For individualism there is no social ethics but only individual ethics.

Universalism, to which we now turn, does not consist in inverting individualism; it does not deny the existence of the individual instead of placing him on the throne. The heart of universalism is rather that it finds the first, primary, original essence from which everything proceeds, not in the individual, but in the totality, in society. From this it follows: (1) that society itself is the true reality; and (2) that the whole is primary, and thus that the individual exists only as a component or member of the whole. The human being cannot develop out of himself but only in spiritual community with others, in mutuality, or polarity—"*Gezweiung*," as Spann calls it. This conception is necessary in order to understand that the world is an intelligible whole where nothing exists without relation to something else and where not blind chance but purpose governs as the final cause. Spann gives many examples—friendship, love, sociability, the family, education, and such—to show how the human mind comes to expression only in

relationship to other minds. He shows that we cannot understand the human mind as a spontaneous growth but only as something created and stimulated into existence, as *lumen de lumine*, where another mind is always the igniting torch, the beaming light out of darkness, the midwife of the mind, to use a Socratic figure.[5] All psychic vitality exists through the being of others in the same way as the organs of the organism are organs through their relations to other members—outside this relation they are not organs but only masses of matter. Psychic or spiritual existence and life means self-existence through existence in the other. The mind is, for universalism, potentiality which can realize itself only in a psychical community.[6] As potentiality, the individual is outside society; potentiality is prior to reality and therefore to society.[7]

The creative actualization of the individual comes about through mutual creative stimulation in mental life—that is, the interchange calls forth, awakes, and literally creates ideas. In the emotional life the phenomenon is like an echo which builds up progressively, as can be seen in enthusiasm, pity, emulation, and shame. The reflection back and forth thus not merely gives occasion for the exercise of the faculty but builds up the faculty itself. If an emotion is aroused for the first time, it at once becomes itself an object of activity. Spann calls this process the stimulation of interests, which is nothing other than the stimulation of the will.

A final factor in the actualization of potentialities is uniqueness. The awakening activity of society is not general or abstract but is always special and concrete. Society or mutuality (*Gezweiung*) constructs itself through the creation of individuality, that is, through the creation of the unique and peculiar. It is also important for the idea of society that in general and predominantly only constructive, spiritual, and moral elements can be actual-

[5] Spann, *Gesellschaftlehre*, p. 11 ff.

[6] Compare for this argument, which is of great importance in Plato's philosophy, B. Landheer, *Der Gesellschafts- und Staatsbegriff Platons* (Rotterdam, 1929), pp. 39 ff.

[7] Cf. Spann, *Kategorienlehre*, pp. 90 ff.

ized. Furthermore, the nature of the individual as a member of society does not interfere with his freedom, because the spiritual community ultimately includes all humanity so that the individual may move from one subgroup to another at will. It is important, however, to keep in mind that in general a larger sized community will mean a looser, more superficial influence on the individuals.

It should now be clear that the totality is something super-individual, something which is beyond and above the individuals, a creative force which does not belong to any single member. The totality, therefore, is a reality on its own account; and so it is most accurate to speak of the objectivity of the community or of the whole. The totality must be thought of in the manner of Plato's "Ideas"—that is, not as a *Realtypus* but as an *Idealtypus*. The whole gives to its members their reality and their structure. The members of the totality, the individual human beings, hold intercourse with each other, not as individuals, but only as co-ordinate members of the whole—that is, again, not directly with each other but only indirectly through the medium of the totality. Just as the individual does not exist in itself, so the universal does not exist in itself. One can never observe totality as a whole itself or as a concrete thing; observable in the concrete are only particular members. It is as if one can only see an arm, a hand, etc., but never the body as such. If totality does not exist as such, it follows that the formula, "Self-existence through existence in the other" becomes "the whole exists in the members."[8] The life of the whole is possible only through the life of the parts; or, in other words, the whole exists through outgrowth in its members (*Ausgliederung*). In this outgrowth (*Ausgegliedertsein*) of a totality (*Ganzheit*) alone lies the reality of the whole (*Ganze*).

What are the consequences of all this for sociology? In the consideration of society as a totality, one must keep distinct two views, the formal and the material conceptions of society. Society is in its essence an organic unity (*Ganzheit*). The heart of this organic unity consists in spiritual community; the life-spring and essence of human society consists in spiritual community. For

[8] *Ibid.*, pp. 55 ff.

this reason society comes to view primarily as realized in partial wholes (*Teilganzheiten*) such as science, art, religion, philosophy, etc. These, one may say, are the subtotalities (*Teilganzen*) of the objective mind. The behavior of human beings is to be understood as a secondary element, which, however, has the quality of actualizing or unfolding the spiritual reality of society. In brief, the formal conception of a society would be, "A society is a spiritual totality (*Ganzheit*) made manifest in action."[9] This formula embodies three points: (1) totality—that is, the most general form or essence of society; (2) spirituality—a further qualification of this totality characterizing it as a non-biological (non-protoplasmic) unity in contradistinction to the animal or plant organism, and as one whose primary character is spiritual; (3) activity—the second attribute of the social totality. Activity should be viewed in two ways: first, in a purely instrumental aspect, as for example in economic activities (economy is a means to an end);[10] second, as the very unfolding or realization of the spiritual entity. It has already been emphasized that organic reality (*Ganzheit*) is revealed in its members. But the whole does not manifest itself in ultimate uniform parts but rather in smaller totalities. The first phase of this self-realization is found in the partial totalities, or as Spann calls them, "Objectification-systems." Society never presents itself to view as absolute society but always exclusively as a determinate form of association or partial totality of society such as the economic system, the state, law, etc.

In what specific partial totalities society will express itself is a question, not to be answered a priori by pure deduction, but through an investigation of the facts, which investigation, however, is based upon an essential intuitive cognition. This question of the subtotalities making up a society belongs to a detailed treatment of social theory and need not be developed here. The

[9] The development of this argument is to be found in *op. cit.*, pp. 62 ff., and in Spann, *Der Schöpfungsgang des menschlichen Geistes*, Vol. I (Jena, 1928).

[10] Spann, *Fundament der Volkwirtschaftslehre*, pp. 25 ff. Spann's complete definition of the economic system is as follows: *"Wirtschaft ist die rangordnungsgemässe Widmung von Mitteln für Ziele durch ausgleichendes und sparendes Abwägen bei Überfülle an Zielen oder Knappheit an Mitteln."*

subject matter of a general social theory is: (*a*) the formal conception of society and the discussion of the significance of the structural laws, or the supreme political principles of society (freedom, equality, justice, etc.), and (*b*) the material conception of society.[11] Thus sociology becomes, first, the theory of all the partial totalities of society and, second, the general theory of the main principles of all the special social sciences. The basic concepts of the special social sciences are thus identical, for they are all taken from general sociology. For certain of the partial totalities, such as science, art, religion, and organization, a general sociology is the only treatment available; for these there are no corresponding special social sciences. Following upon general sociology come the special social sciences, such as economics, demography, theoretical statistics, and descriptive statistics, ethnology, history, jurisprudence, culture anthropology, and finally the practical arts or different branches of technology (accounting, marketing, home economics, etc.).

This whole structure of the social sciences is ruled by a procedure which is applicable to all. In the first place the social sciences are sharply differentiated from the natural sciences; for a social science the main category instead of causality is organic function. Spann proves this in detail for all the social sciences. It may be sufficient here to use a single example, which may be drawn from statistics;[12] parallel curves, for example, between the price of wheat and criminality do not show any causal connection between the price of wheat and crime, as it is generally believed. This correlation can be understood only by considering the economic system as a whole and by understanding how the higher price of wheat becomes the motive for a particular mode of behavior for particular persons. The reason for the phenomenon can be understood as logically and intuitively comprehensible, never as a cause in a mechanical sense. As this argument is valid for all the social sciences, we can summarize thus: the task of the social sciences is to investigate the organic relationships of members involved in social phenomena. In the social sciences the only methods admissible are those determined by relations between

[11] Cf. Spann, *op. cit.*, pp. 249 ff. [12] Cf. *Ibid*, pp. 54 ff.

whole and part, and hence these methods have for their objective, first, to determine what totalities are met with in their fields of study, that is, to characterize, and to distinguish them from other totalities. Out of this task develops the second, namely, to determine the organic relationships (*Gliedlichkeitsart*), that is, to define the relation between members and whole, since the member always exists in relation to a higher whole, namely, society in its widest sense. On this basis the third aim is to describe the structure of the totality in terms of its partial or subtotalities down to the last members in the individual existence and to discover the laws of relationship which integrate the parts into the whole. The way in which Spann carries out this task for sociology cannot be shown in detail in a short exposition of his general principles.[13]

Finally it may be worth while to restate in a summary the relation between totality and part. Totality exists—and this is the basic fact and starting point; it is category in an ontological and not in an epistemological sense. In order to understand fully this conception of totality one must be clear as to the way in which Spann conceives the relation between the "Is" and the "Ought."[14] In opposition to most modern philosophers Spann holds that existence and value (the "ought," the valid, the norm) are determined in the most intimate relationship to each other. The conception of existence involves not merely how a thing is but how a thing should be. Spann completes the above-quoted sentence of Aristotle, "The whole necessarily precedes the part," by the state-

[13] In *Gesellschaftslehre* Spann calls his system "kinetic universalism." The main principle of his system is that the realization of the totality implies at the same time its change. This dynamic universalism could be related historically in the first place with a sociological theory of Plato's idealism, as was tried recently by E. K. Winter in his book, *Platon, Das Sociologische in der Ideenlehre* (Vienna, 1930). Spann himself regards as representative of universalism, Plato, Aristotle, the Stoic philosophy, the Chinese social philosophy of Confucius and of Lao-tse, the social philosophy of Christianity and of German Idealism, of Fichte, Schleiermacher, Schelling, and the Romantic school, particularly Adam Müller.

As members of Spann's school may be named: W. Andreae, *Grundlegung einer neuen Staatswirtschaftslehre;* J. Baxa, *Gesellschaftslehre* (Leipzig, 1928); Karl Faigl, *Ganzheit und Zahl* (Jena, 1928); Walter Heinrich, *Grundlagen einer universalistischen Krisenlehre* (Jena, 1928), and others.

[14] Cf. the chapter on methodology in Spann's *Gesellschaftslehre*, pp. 533–71.

ment, "The perfect necessarily precedes the imperfect." The totality is thus a higher order of reality, higher than the existence of the members. In the comprehension of the totality one gains at the same time an objective standard of value. The existence of imperfection is not always clearly explained in Spann's system. He derives imperfection from nonessential actualization of the totality in the parts; but how this in turn comes about remains an open question.

The main consequence of his system is that all the social sciences immediately become normative and evaluative sciences. One cannot really understand how a thing *is* without understanding how a thing *should be*. This leaves open the cardinal question, which is not touched by Spann, whether or not knowledge of the nature of things in this sense is possible at all. The slighting of this question in Spann's philosophy carries with it the danger of assuming equality of the human and the absolute minds. For since the human mind is capable of understanding exhaustively every totality, it must be able to understand the absolute itself. This question raises the problem of whether or not knowledge of a certain subtotality, for example, the economic system, would be possible without knowledge of the Absolute (*Urganzheit*). It must be remembered that knowledge of the subtotalities is not drawn from experience but through deduction from the higher totality. Whether this intuitive knowledge is possible in the realm of nature is left in doubt by Spann, although in his *Kategorienlehre* he inclines to this assumption. Regarding society, however, he does not so much as raise the question. It seems to the writer a fundamental truth that the human mind in understanding social facts judges them. It would be interesting to try to prove that even where valuation is denied it is nevertheless present. It is very important that in the social sciences Spann deliberately makes the shift to the normative conception of science and carries out the idea in the most consistent manner.[15]

[15] See Rickert, *Kulturwissenschaft und Naturwissenschaft* (Tübingen, 1921), for the difference between physical and value sciences.

See Spranger, *Sinn der Voraussetzungslosigkeit der Wissenschaft* (Berlin, 1929), for the determination of science by the different types of culture.

It remains true however that men will never agree as to what is right. The question as to the true or ideal state is unescapable; but any effort to answer it exactly and in detail takes one into the field of dogmatism. It seems important, however, in a period in which scientific knowledge is overestimated to stress the importance of valuation; it has never been possible to draw a clear line between fact and value. Ultimately it is the fruitfulness of a scientific principle which tests its truth. The point for Spann's sociology is the intimate dependence of his methodology on the conception of totality. The function of the member or part of totality can only be understood in relation to totality, hence only so far as the latter itself is comprehended. It would be difficult to find an objection, however, against the use of inductive knowledge to prepare the way for and to complete the rational comprehension visualized by Spann. That the idea of functionality is very valuable is proved by Spann's *Fundament der Volkwirtschaftlehre*, which opens new and interesting perspectives in this branch of the social sciences.

Whether the functional concept (*Leistungsbegriff*) can be used also for the physical sciences is a question which must be left open as must the validity of the totality-member relationship in the general realm of nature. As soon as the idea of totality is used too broadly it becomes a mere class or genus concept in contrast with concrete reality. Probably this would not be in accordance with the original intention of Spann. A further discussion of these problems, however, would be purely philosophical—as in general the whole tendency of Spann's social theories is strongly toward philosophy. It may be regarded, however, as a sign of the importance of these ideas that one cannot deal with them without coming to the most ultimate and profound problems that challenge the human mind.

BARTH LANDHEER

ROTTERDAM

MEASURES OF AVERAGE ELASTICITY OF DEMAND[1]

THE measure of elasticity of demand developed by Alfred Marshall may be applied only to individual points on a demand curve. The measures of elasticity suggested in this article represent an extension of the Marshallian idea to any given arc of a demand curve. As a matter of fact, there are many possible measures of arc elasticity, depending upon (1) the kind of average employed and (2) the system of weighting.

An "unweighted" average of arc elasticity is indeterminate. Let $P_0, P_1, \ldots, P_{n-1}$ be any series of points at short intervals along the arc P_0P_n of a given demand curve. Suppose it is desired to find the arithmetic mean of the elasticities of $P_0, P_1, \ldots, P_{n-1}$, each weighted equally. Let this average elasticity be designated by E. Roughly expressed,[2]

$$E = \frac{-\frac{\Delta x_0}{x_0}\Big/\frac{\Delta y_0}{y_0} - \frac{\Delta x_1}{x_1}\Big/\frac{\Delta y_1}{y_1} - \cdots - \frac{\Delta x_{n-1}}{x_{n-1}}\Big/\frac{\Delta y_{n-1}}{y_{n-1}}}{n}.$$

If all the increments in x and y be allowed to approach zero, and the number of points increases accordingly,

$$E = \frac{\int -\frac{ydx}{xdy}}{\infty} = \frac{\infty}{\infty}.$$

E, on the basis of the given information, is indeterminate. There are many possible series of points at short intervals along the arc

[1] The original manuscript of this article was read by Professor Henry Schultz, of the University of Chicago, and by Professor F. C. Mills, of Columbia University. Several helpful suggestions which they offered have been embodied in the text. The author himself, however, assumes entire responsibility for any errors.

[2] It is assumed that the increments are measured from x_0 to the right. For the points $P_n, P_{n-1}, \ldots, P_1$, a similar result may be obtained with increments measured from x_n to the left.

P_0P_n, and the equally weighted average elasticity of each of them approaches a different limit.

It is possible, however, to introduce several systems of weighting in the calculation of the arithmetic mean of elasticity of any such series of points. For example, the elasticity of each point in the series may be weighted by (*a*) the absolute increment in x, (Δx),

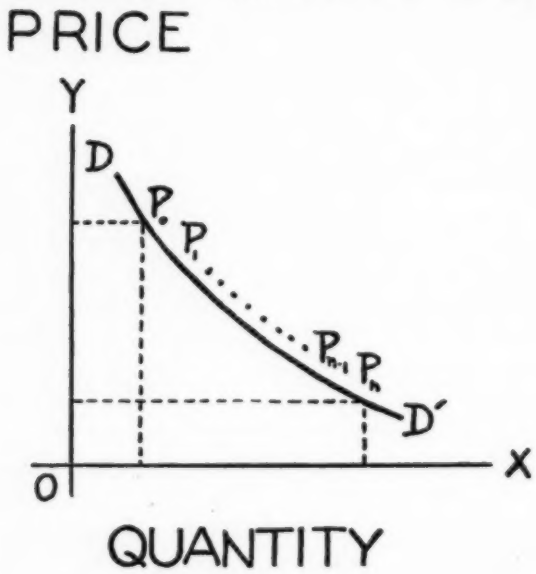

FIG. 1

(*b*) the absolute increment in y, (Δy), (*c*) the relative increment in x, $\left(\frac{\Delta x}{x}\right)$, (*d*) the relative increment in y, $\left(\frac{\Delta y}{y}\right)$, (*e*) the product of x and Δy, $(x\Delta y)$, (*f*) the product of y and Δx, $(y\Delta x)$, or (*g*) the increment in the arc $\left(\sqrt{\Delta x^2+\Delta y^2}\right)$. As a result of weighting by any one of these methods, it will be found that the average elasticity of any series of adjacent points on the arc, can, in theory at least, be exactly determined. Each system of weights, however, will result in a somewhat different average.

The average obtained by weighting the elasticity of each point

in any adjacent series on the arc by $x\Delta y$ can be expressed by means of a simple formula, and is capable of simple graphical representation. Roughly expressed,

$$E=\frac{-\frac{\Delta x_0}{\frac{x_0}{\Delta y_0}}\cdot x_0\Delta y_0-\frac{\Delta x_1}{\frac{x_1}{\Delta y_1}}\cdot x_1\Delta y_1-\cdots\cdots-\frac{\Delta x_{n-1}}{\frac{x_{n-1}}{\Delta y_{n-1}}}\cdot x_{n-1}\Delta y_{n-1}}{x_0\Delta y_0+x_1\Delta y_1+\cdots\cdots+x_{n-1}\Delta y_{n-1}}.$$

If the increments approach zero, and the number of points increases accordingly,[3]

$$E=\frac{\int_{x_0}^{x_n} y\,dx}{\int_{y_0}^{y_n} x\,dy}.$$

[3] As this formula stands, it requires two integrations. It will not always be possible to perform both integrations directly. Either integration, however, may always be derived from the other. According to Figure 2,

$$A_0P_0P_nA_n=B_0P_0P_nB_n+OB_nP_nA_n-OB_0P_0A_0.$$

$$\int_{x_0}^{x_n} y\,dx=\int_{y_0}^{y_n} x\,dy+x_ny_n-x_0y_0.$$

Conversely,

$$\int_{y_0}^{y_n} x\,dy=\int_{x_0}^{x_n} y\,dx+x_0y_0-x_ny_n.$$

If $y=f(x)$, by substitution,

$$E=\frac{\int_{x_0}^{x_n} f(x)\,dx}{\int_{x_0}^{x_n} f(x)\,dx+x_0y_0-x_ny_n}.$$

If $x=F(y)$,

$$E=1+\frac{x_ny_n-x_0y_0}{\int_{y_0}^{y_n} F(y)\,dy}.$$

If this formula in any of its forms is applied to any segment of a straight-line demand curve, it will be found that E is equal to the elasticity of the mid-point of the segment.

Any measure of elasticity of demand, however, is a ratio. The logical average to use in connection with ratios is not an arithmetic mean but a geometric mean. Elasticity may vary in one direction from unity upward indefinitely; but, varying in the other direction, it cannot under any circumstances be less than zero. An arithmetic mean is not a fair average of such variations.

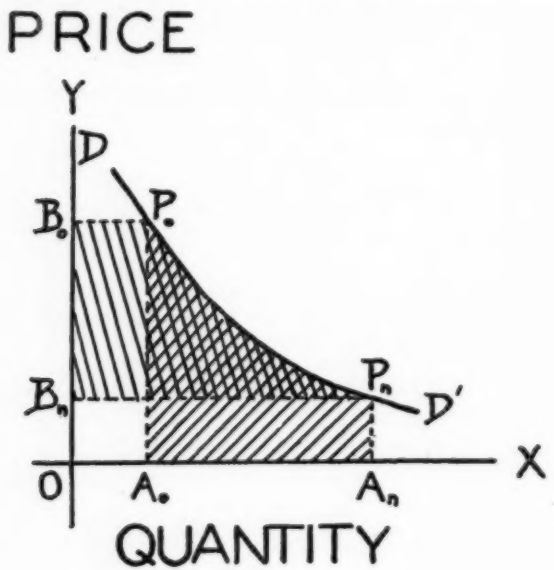

FIG. 2

$\int_{x_o}^{x_n} y dx$ equals the area $A_oP_oP_nA_n$ shaded ■.

$\int_{y_n}^{y_o} x dy$ equals the area $B_oP_oP_nB_n$ shaded ■.

E is the ratio between these two areas.

It has been shown that the "unweighted" arithmetic mean of arc elasticity is indeterminate. In similar manner, it may be shown that the "unweighted" geometric mean is also indeterminate. Particular weighted geometric means, however, are determinate.

A general formula for a weighted geometric mean of n terms may be expressed as follows:

$$M_g = \sqrt[w_1+w_2+\cdots+w_n]{(a_1)^{w_1}\cdot(a_2)^{w_2}\cdot\cdots\cdot(a_n)^{w_n}}.$$

Each term is raised to a power corresponding to its weight, and a root corresponding to the sum of the weights is extracted from the product.

A wide variety of choice is possible in selecting a system of weights for the particular geometric mean under consideration. The elasticity of each point in the series may be weighted by any of the elements pointed out in connection with the arithmetic mean. It is suggested, however, that the elasticity of each point be weighted by Δy. Elasticity of demand is defined as the responsiveness of the amount demanded to changes in price. This implies that $x = F(y)$. It seems reasonable to lay special emphasis on the change in the independent variable. On this basis, the geometric mean of elasticity of any series of points at short intervals on a given arc may be roughly expressed as follows:

$$E = \sqrt[\Delta y_0+\Delta y_1+\cdots+\Delta y_{n-1}]{\left(-\frac{\frac{\Delta x_0}{x_0}}{\frac{\Delta y_0}{y_0}}\right)^{\Delta y_0}\cdot\left(-\frac{\frac{\Delta x_1}{x_1}}{\frac{\Delta y_1}{y_1}}\right)^{\Delta y_1}\cdots\left(-\frac{\frac{\Delta x_{n-1}}{x_{n-1}}}{\frac{\Delta y_{n-1}}{y_{n-1}}}\right)^{\Delta y_{n-1}}}.$$

$$\log E = \frac{\Delta y_0 \log\left(-\frac{\frac{\Delta x_0}{x_0}}{\frac{\Delta y_0}{y_0}}\right)+\Delta y_1 \log\left(-\frac{\frac{\Delta x_1}{x_1}}{\frac{\Delta y_1}{y_1}}\right)+\cdots+\Delta y_{n-1}\log\left(-\frac{\frac{\Delta x_{n-1}}{x_{n-1}}}{\frac{\Delta y_{n-1}}{y_{n-1}}}\right)}{y_0-y_n}.$$

If the intervals approach zero, and the number of points increases accordingly,

$$\log E = \frac{\int_{y_n}^{y_0}\left[\log\left(-\frac{ydx}{xdy}\right)\right]dy}{y_0-y_n}.$$

Substituting $x=F(y)$ and $\frac{dx}{dy}=F'(y)$, we may also obtain:

$$\log E=\frac{\int_{y_n}^{y_0}\left[\log\left(-\frac{yF'(y)}{F(y)}\right)\right]dy}{y_0-y_n}.$$

In either form, the formula makes possible a general measure of arc elasticity. It may be most easily applied, when, in accordance with the suggestions made by Professor H. L. Moore, one of the following assumptions is made:

$$-\frac{ydx}{xdy}=C$$

$$-\frac{ydx}{xdy}=A+By$$

$$-\frac{ydx}{xdy}=A+By+Cy^2.$$

With the aid of a table of integrals, however, it may also be applied to any of the other functions in common use.[4]

The present article presents only the theoretical side of the measurement of elasticity, but it is written with a practical purpose. It has been customary, when a comparable measure of elasticity of demand is desired, to fit to the data a curve of the constant-elasticity type,[5]

$$\log x=A+B\log y,$$

even though some other function is more suitable from other points of view. The formulas in this article are suggested not only as more accurate measures but as labor-saving devices. The

[4] If elasticity of demand is calculated on the basis of finite changes, two different averages may be obtained: (1) moving up the curve, (2) moving down the curve. This difference, however, will disappear if the measure is based on infinitesimal changes.

[5] The lead in this direction was taken by R. A. Lehfeldt. See his article, "The Elasticity of Demand for Wheat," *Economic Journal*, June, 1914, pp. 212 ff. For a criticism of Professor Lehfeldt's method from another point of view, see Henry Schultz, *Statistical Laws of Demand and Supply* (1928), pp. 211–12.

writer has found that with segments of a demand curve of the type $x=a+by$,

$$\text{Formula I,} \qquad E=\frac{\int_{x_n}^{x_0} y\,dx}{\int_{y_n}^{y_0} x\,dy},$$

gives approximately the same results as the more theoretically accurate

$$\text{Formula II,} \qquad \log E=\frac{\int_{y_n}^{y_0}\left[\log\left(-\frac{y\,dx}{x\,dy}\right)\right]dy}{y_0-y_n}.$$

The reason is that Formula I gives more weight to the lower portion of the segment where the elasticity from point to point is less than on the upper portion. This particular arithmetic mean tends to coincide with the geometric mean. The result given by Formula I when applied to $x=a+by$ is always the elasticity of the mid-point of the segment. It is suggested, therefore, that when $x=a+by$ is found to be the best fit to given data on demand, the elasticity of the point on the line midway between the limits of observation be taken as a measure of the elasticity of demand for the commodity in general. It is believed that this simple device is more accurate than the fitting of

$$\log x=A+B\log y.$$

When a non-linear relation exists, the elasticity of the mid-point is no longer of such significance. In such a case, however, Formula II will give a measure of elasticity with less labor and trouble than a specially plotted $\log x=A+B\log y$. If the elasticity tends to vary directly with the price, and inversely with the quantity, Formula I may be used. It is easier to calculate than Formula II. Under the conditions specified, it may be expected to give approximately the same results. The important thing is to draw conclusions regarding elasticity from the curve which best fits the data.

A. J. NICHOL

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BOOK REVIEWS

Die drei Nationalökonomien. By WERNER SOMBART. München und Leipzig: Duncker & Humblot, 1930. Pp. xii+352. $3.75.

On certain rare occasions a scholarly treatise appears from the pen of a scientist, which presents a veritable cross-section of human knowledge. Such a treatise is *Die drei Nationalökonomien* by Werner Sombart. The title conveys only very inadequately the profundity of erudition revealed in this modern encyclopedia of economic thought.

The author begins his analysis by deploring the loose and careless use of terminology in connection with supposedly scientific economic studies, subjecting the words "economy" and "economics" to a thoroughgoing scrutiny at the outset. This survey of basic concepts is fundamental to a careful delimitation of the sphere of human activity to be investigated. Much of the confusion in so-called "economic" thought arises out of the failure to understand concepts clearly. And perhaps the greatest confusion centers around the word which has been chosen to designate a field of scientific endeavor, namely, economics.

The ease with which compound words are concocted in the German language, moreover, has contributed to no small degree to the multifarious and at times indefinite usage of terms, which the scientist must subsequently dissect and delimit. The need for painstaking analysis of economic terminology is clearly recognized by the author. He views with a certain degree of mistrust the precision and clarity with which, for example, the English language is able to differentiate between science and art (p. 325). Thus he quotes John Stuart Mill: "*Science* is a collection of truths, *art* a body of rules or directions for conduct. The Language of Science is: this is or this is not; this does or does not happen. The Language of Art is: do this, avoid that."

"These sentences are so clear," says Sombart (p. 325) "that we Germans cannot suppress a slight mistrust, that something may not be in order. . . . . But occasionally something may be clear and, in spite of that, be correct." This attitude of mind pervades the entire discussion, and it is questionable whether the Sombartian laboratory method of dissecting economic concepts is always conducive to further clarification of thought.

In his endeavor to bring a certain degree of orderliness into the chaos of economic and pseudo-economic thought, the author divides and classifies "economic" knowledge into three "schools" or "systems." The first he calls *die richtende Nationalökonomie*, which might be translated the "moralizing or deterministic political economy." Its representatives strive to interpret economic phenomena in terms of the eternal order of things, the divine ruling, or the immutable laws of nature. The consciousness of a "right" economy, a "just" price and the like pervades the so-called economic thought of the representatives of the *richtende Nationalökonomie*. Some discover the "right" order of things in divine decree (cf. Scholasticists), others in the eternal laws of nature (cf. Adam Smith), and still others in processes of pure reason or enlightenment (cf. Karl Marx). But all such explanations and interpretations of economic phenomena are, according to Sombart, philosophical and metaphysical rather than scientific.

The second system is designated the *ordnende Nationalökonomie* or what may be called the "classifying political economy." Following the methodology of the natural sciences the representatives of this "school" endeavor to formulate general economic laws, based on careful observation and analysis of individual phenomena. The laboratory methods employed in the study of the natural sciences are used by the representatives of the *ordnende Nationalökonomie*, who strive to break up their subject matter into its component parts and, subsequently, to establish the universal applicability of the knowledge they have gained from their individual analyses. Even though their findings may not be universally intelligible, any more than is the Einstein theory of relativity, they are, nevertheless, universally applicable, when properly qualified.

Their procedure is scientific, but utterly inadequate as a complete explanation of "political economy" as Sombart interprets it. Hence the third "school" or "system" of political economy, with which the author identifies himself, namely *die verstehende Nationalökonomie* or the "understanding political economy," which represents the endeavor to fathom, within the limitations of human knowledge, the meaning of economic activity in its relations to all phases of human activity, and ultimately the meaning of life itself. Neither the laboratory method of the natural scientist nor the moralizing method of the philosopher is adequate to provide a full explanation of the science of economics. A combination of both theory and practice is essential to a full understanding. The reviewer is fully in accord with the author when he re-

marks *"Wer nicht Theorie und Empirie gleichmässig treibt, ist überhaupt kein ganzer Nationalökonom, sondern nur ein Teil von einem solchen"* (p. 319).

The student of the history of economic thought will discover in this latest work of one of the great living economists a veritable storehouse of stimulating, critical analyses of economic doctrines.

KARL W. H. SCHOLZ

UNIVERSITY OF PENNSYLVANIA

The History of Economics. By OTHMAR SPANN. Translated from the nineteenth German edition by EDEN and CEDAR PAUL. New York: W. W. Norton & Co., 1930. Pp. 9+328. $3.50.

Every teacher of economics should enthusiastically welcome this translation of Spann's well-known *Haupttheorien der Volkswirtschaftslehre*, and should urge every student to read it thoughtfully. Professor Allyn Young prescribed it for his classes in the history of economic thought, even when it was available only in the cheaply printed German edition in a popular series, and the reviewer has been given to understand that he was instrumental in getting it translated. If scholarly production were governed by the higher economics of need instead of that of carrying coals to Newcastle, it would have been given to the Anglo-Saxon world in the first place. After such a general statement it is needful to be more explicit, and in being explicit it will be natural to be somewhat more critical.

In a field in which controversy in the deepest sense, the clash of philosophies of knowledge and of value and social policy, is the life-blood of the discussion, it is perhaps not to be expected that anyone will write a history of doctrine in a tone of impartiality; or, if it is so done, that it will be more than a perfunctory performance. In any case, such is the indication of results to date. Hence the student finds himself in a dilemma. He cannot get the history of economic thought from any one controversial book, or from any number made by the scissors-and-paste method. Hence he must read more than one book, of the controversial sort and from different points of view, of course. Some three or four would be necessary really to map the field. One would of course be written by an author who believed in the "classical" system and defended it—far better explicitly than unconsciously. Another would attack this from the socialistic angle, but that is not

really so very divergent, where the same subject matter, value theory, is in question. Another, perhaps, would attack it from the new behavioristic standpoint, the conception of economics in terms of empirical facts and laws or commodity statistics. But it is not clear how the history of dogma would be developed from this point of view specifically; nor is there any danger that the student of today will neglect the point of view itself. The really important contrasting viewpoint is that of the culture history or "institutional" or social-psychological interpretation and explanation. Of several possible species under this genus the "universalistic" economics of Spann, based on the literary and philosophic world-view of German Romanticism, is undoubtedly the most effectively thought-provoking. As no reader will need to be told, the frankness of the author's controversial tone leaves nothing to be desired.

For a really understanding presentation and criticism of the classical position, it goes without saying that no one should be directed to Spann—any more than to an out-and-out advocate. Much of what the book says about what we call "orthodox" economic theory is quite unessential or even false to the inner logic of the system itself; but that is likely to be the case where it would be easiest to cite chapter and verse from its own exponents. For example, no difficulty is involved in going through the "Critique of the Doctrine of Marginal Utility" (pp. 264–69) and showing that to every statement in the attack there is a retort conclusive from its own point of view. A typical instance is the author's notion that the principle of final increments is invalidated by complementary relations among commodities (or productive services) in use. This is not true, but how many "orthodox" textbooks face the problem and show in clear and general terms why it is not? Again, on page 267 we read: "The total available supply of a commodity does not acquire its value through the summation of the individual utilities . . . . nor yet through our multiplying the marginal-utility value by the number of pieces; it acquires its value only as an effluence from the more exalted wholes of which it is a member." The first half of the sentence is either true or false, depending on the definition of "value," and in neither sense is it contradictory with the second half, as the author implies.

Particular mention should be made of the political doctrine of fascism, which Spann affects to deduce (not in this book) from his organic economics and sociology and for which he has become "notorious," among those who dislike it. There is no necessary or logical

connection whatever between the two, but this, interestingly enough, is a feature which Spann's teaching has in common with all its competitors. It is a common observation that the fatalism of the "materialistic interpretation" does not gibe well with the revolutionary program of Marxism, and the same remark applies to the simultaneous stress on "scientific law" and on "control" of the current statistical school (since in society controller and controlled are identical and the predictability which applies to the one must apply to the other). In the classical tradition, there is more conflict than consistency between the hedonistic theory of value and the political doctrine of free competition, meaning as it does the freedom of the individual in a position of advantage to use his power to increase his advantage indefinitely. It seems, to the present writer at least, that in so far as Spann's principle of *geistliche Ganzheit* is real, practical social problems do not arise, and the consistent social philosophy would be that of anarchism.

Whatever the merits or these comments, the educable student of Anglo-Saxon background who reads Spann should be helped in grasping the idea that there is another psychology, another sociology, and another ethics than those traditional in his own culture, in terms of which the relations of buying and selling, hiring and firing, leasing, lending etc., of which economics treats, look very, very different. If he does this, and thinks himself into some comprehension of the relations between presuppositions and superstructure in the two most sharply contrasting economic systems, and goes on to a critical comparison of different possible presuppositions, he will have obtained an especially effective introduction to the basic problems of social and human phenomena.

The translation is obviously the work of persons of literary competence not too familiar with the special literature of the subject. It is unfortunate, for example, to translate *Zurechnung* by the word "accounting" (p. 260) when the much-better term "imputation," used to translate this German word a generation ago, has been standard English for the concept ever since. But glaring examples of the sort are fairly rare, and in general the style is clear and readable, if not conformable to the usual jargon.

FRANK H. KNIGHT

UNIVERSITY OF CHICAGO

A Discourse of Trade, from England unto the East Indies, 1621. By THOMAS MUN. Reproduced from the first edition by the Facsimile Text Society. New York, 1930. Series V: "Economics," Vol. I. Pp. 58. $1.50.

This new venture in the reproduction of scarce economic texts is an admirable one. The reproduction is by a photographic process, which is assurance, of course, of accuracy of text, and at the same time is economical. The present text is adequately clear for all purposes, and will serve as well as would an original copy all the needs of the economist. The prices of these reprints are to be quite low, and it seems important that economists support this laudable enterprise by subscribing for the economic series. It is to be hoped, however, that the Facsimile Text Society will concentrate on the reproduction of texts which are really scarce and unavailable in the ordinary good library. Mun's *A Discourse of Trade*, while exceedingly rare in the original, was reprinted in one of McCulloch's collections of early economic tracts, as well as in Purchas' *Pilgrims*. Some of the other economic items announced as forthcoming are even less scarce, as for instance, Locke's economic essays, which are accessible in all the many editions of Locke's works, as well as in separate reprints. It is worth while to have accurate reproductions at so low a cost of even these works. But unless there has been substantial change in the text of later reprints or editions, it would seem wiser to devote the available funds, at first at least, to those items which are really inaccessible at present, in any form, to all but a privileged few. There are exceedingly rare and yet highly important works by Malynes, Misselden, Potter, Robinson, Barbon, and Tucker, to specify only a few, which in my judgment have superior claims over Locke's essays to reproduction, if not for their greater importance, at least because of their greater rarity whether in the original or in reprints. But to Seligman, Hollander, and the others who are giving aid to this project, all those who are interested in the historical origins of our modern economic thought owe a great debt of gratitude.

JACOB VINER

UNIVERSITY OF CHICAGO

Logik des Geldes. By BRUNO MOLL. Third edition. München: Duncker & Humblot, 1929. Pp. xvii+104. M. 4.50.

Although this little book, first published in 1916, has managed to reach its third German edition, there is no evidence that it has as yet attracted the attention of non-German workers in the field of monetary theory. If one were not so acutely conscious of the part which linguistic barriers play in bringing about the unfortunately large amount of insularity which characterizes current work in economics, one might be inclined to attribute the lack of interest in this book outside of Germany to its own almost exclusively German orientation. It is characteristically German in its choice of problem and shows a direct acquaintance only with the German literature. (The single reference (p. 31) to a non-German work—H. D. Macleod's *Theory and Practice of Banking*—lists Macleod as a metallist: a curious characterization, surely, of the writer whose treatment of the relation between money and the idea of a "debt" represents the principal adumbration of Mr. R. G. Hawtrey's nominalist variations on the theme that "the idea of money is derived from the idea of a debt"!)

None the less, it would be doing this book a serious injustice if one failed to recommend unreservedly to all students of the subject a careful study of the first quarter of the book, which is really a discussion of the epistemology of monetary theory—or, more accurately, of that part of monetary theory which the reviewer has ventured elsewhere to call "monetary metaphysics," and which is concerned with such problems as that of money's "essential nature," its formal definition, and the "functions" it performs. Such a study is called for, in the first place, by considerations of fairness to German monetary theory of the last two decades. The vigor with which the author belabors his opponents in controversy ought not to blind one to the fact that here, at least, is evidence that not all German writers on the "nature of money" were content to imitate the dogmatic arrogance of writers like Knapp and Bendixen on the one side, and their metallist opponents, on the other. Here, also, is evidence (see, e.g., pp. 12 ff.) that a primary interest in the problems of "monetary metaphysics" does not necessarily involve the suggestion that these are "the" problems of monetary theory, or that the so-called "quantitative-dynamic" problem of the determination of the "value" of money is of relatively slight importance. A study of these first thirty pages is recommended to American and English readers, in the second place, as a prophylactic against infection from the dogmatism which may yet arise in connection with those problems

of "monetary metaphysics" which it is one of Mr. Hawtrey's many distinctions to have re-introduced into the English literature on monetary theory.

The remaining three-quarters of the book, which are concerned chiefly with what the author calls the "Problem of the End," are less heartily recommended. One can admit the "originality" which the author claims (p. ix) for this mode of approach to the problem of the "nature" of money: indeed, if the author is to be criticized at all in this connection, he is to be criticized for having been altogether too generous in his rhapsodic tribute (pp. 51 ff., and especially p. 56) to such a writer as Adam Müller, whose misty speculations he regards as representing a "significant attempt at solution" of the problem. The reservations one has with respect to the "Problem of the End," as a conceptual device, concern rather the probabilities of its further usefulness as an analytical tool. Yet, even here, one is inclined to refrain from adopting a hostile attitude. After all, it is by means of the "Problem of the End" that the author arrives at his "reconciliation" (pp. 91 ff). of metallism with nominalism; and the demonstration of the possibility of such a "reconciliation" is in itself a welcome contribution, even though one could wish that the reconciliation were effected in connection with some problem of more realistic content. Most disarming of all, however, is the consideration that the author has himself provided, in the first quarter of his book, all necessary warnings against those claims to a "final" truth within this branch of monetary theory which alone would justify a violently hostile attitude toward a really suggestive and significant approach to complicated problems.

ARTHUR W. MARGET

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The Recovery of Germany. By JAMES W. ANGELL. New Haven: Yale University Press, 1929. Pp. xix+425. $4.00. Also published in German translation under the title *Der Wiederaufbau Deutschlands, Von Versailles bis zum Young Plan.* München: Duncker & Humblot, 1930. Pp. xvi+323.

Professor Angell has done here a remarkably fine piece of journalism based on extensive investigations carried through in less than a year. The book is written largely for persons not specialists in economics. Economists will read it with as much profit as others; but Angell has wisely eschewed difficult and controversial issues which might confound his wider audience—and without serious sacrifice.

The author outlines the terms of the Treaty of Versailles and tells concisely the story of Reparations. He covers the essentials of German economic history since the war, with emphasis upon the years after 1923. He deals convincingly with the major industries, cartels and trusts, foreign trade, the capital market, and with the future of the German economy. His conclusions display a strange mixture of diffidence and assurance; and what appear to be convictions are often qualified almost to contradiction at other points. This, however, is perhaps inevitable.

Angell has brought together a mass of valuable statistical material. In this task he has had competent assistance and unusual co-operation —all of which he adequately acknowledges. Moreover, he has tempered his inferences and conclusions through consultation with the best-informed persons in Germany. His contacts, indeed, have been so wide that the book may be read as a critical synthesis of competent German opinion in numerous fields.

Careful readers will find occasion frequently to dissent and demur. The remark (pp. 24, 35) that an interest rate of 20 per cent daily is equivalent to 7,300 per cent annually involves an error of astronomic magnitude. It is not a cautious statement (p. 36) that, when the Government in 1921 "imposed a flat levy of one-tenth of the harvest, ... the agriculturalists blandly raised the prices of the remaining nine-tenths. . . . ." Nor is one convinced that (p. 58) "The distribution of wealth and income was violently altered, and in a way that greatly reduced the general buying power of the German people." It is hardly obvious that "The high price of foods in turn raises the cost of living of the working classes, and thus entails higher money wages" (p. 253). After being told of the remarkable progress in management and technology, one is reluctant to believe that (p. 272) "The rise in money wages . . . . by 80 per cent [1924–28] is due almost entirely to their [trade unions'] efforts."

There is repeated emphasis upon the consequences of low-cost capital-installation during the Inflation. It is reasonable to hold (pp. 156, 160) that this has retarded elimination of inefficient firms; but one is perplexed by the contention (pp. 48, 58, *passim*) that such past events served to increase the competitive power of German industry in world markets after 1923. The fact of addition to capital equipment is important; but what it cost or what indebtedness remained against it should hardly affect the volume of exports.

In passing one may question Angell's view that equipment installed

during the Inflation was mostly of poor quality, obsolete design, and short life. There is truth in the contention but probably exaggeration too—and some inconsistency. Indeed, one suspects that the whole picture of change since 1923 has been somewhat overdrawn. It seems probable—and this view is not original with the reviewer—that the fundamental structure of German industry at the end of 1923 was far stronger than hasty inference from statistics or reliance on distinguished opinion might lead one to believe.

Angell gives judiciously meager attention to the so-called transfer problem. His emphasis upon the facts of Germany's foreign-trade position may afford comfort to proponents of less orthodox views; but he appears to see clearly that the real problem is one of governmental policy and political stability. Thus, he warns (pp. 329, 330) against the confusion implicit in the notion that Reparations have been paid merely out of foreign loans; and he finds cause for alarm in the fiscal practices of the Reich and in the wage-policies of trade unions and regulative agencies. However, there are statements—perhaps unguarded ones—which might suggest a different interpretation. For example, he says:

> The cartel system, insofar as it is applied to exports, necessarily impedes price reductions; the markets, in terms of their capacity to absorb additional physical quantities of goods, are restricted (that is, present demands appear to be very inelastic); and in view of the high fixed charges many companies carry, this presumptive narrowness of the demand makes it probable that reducing prices would lead only to a net loss [p. 186].

Such passages invite many criticisms; but one doubts especially if Angell would subscribe to the implications for the transfer-problem.

Angell seems to regard capital imports much as Senator Grundy regards imports in general. He speaks (p. 301) of "that eventual independence of further capital imports which is the cardinal necessity for her [Germany's] future economic life." This reflects a prevailing view. Yet no one denies— certainly not Angell—that foreign loans have contributed enormously to the progress of German industry nor that the high interest-rates reflect high capital-productivity. Of course, there is something intriguing about the lender-status; but the inadequacy of domestic savings at ordinary interest rates may signify strength as well as weakness—as one realizes upon comparing the circumstances of Germany and England.

To view with alarm Germany's balance of trade and her position in the markets for loan funds is to conjure up a new transfer-problem

to displace one which, having served well in a worthy cause, deserves now only to be forgotten. The new "problem" is obviously one of gradual, long-run adjustment; and it is hard to find basis here for distrust of a market economy in which such things are handled pretty well without deliberate management. Persons worried about Reparations will do well to concentrate attention upon the fiscal policy of the Reich and let German borrowers look after their own affairs.

Adverse criticism of this kind hardly touches the basic fabric of information and interpretation which Professor Angell has woven. It relates largely to the asides and "continuity" in a story competently told and, however pointed, has little importance for judgment of the book as a whole. The author has achieved a narrative which reads nicely, which informs abundantly, and one which will contribute in an important way toward international understanding.

H. C. SIMONS

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Aussenhandel und Aussenhandelspolitik. (Die internationalen Wirtschaftsbeziehungen.) By FRANZ EULENBURG. Abteilung des "Grundriss der Sozialökonomik," Vol. VIII. Tübingen: J. C. B. Mohr (Paul Siebeck), 1929. Pp. viii+327. M. 20.

This is the most recent and most comprehensive German monograph on international trade. It comprises not only the theory of international trade but also a history of international economic relations, a theory and history of commercial policy, and a description of all kinds of institutions and regulations affecting and dealing with international trade. It treats the monetary aspects of the subject, as well as the underlying fundamental relations. Nor does it omit a sociological analysis of the attitude of political parties toward international economic policy. There is no aspect of international economic relations which does not find due consideration in a special section or chapter: "Technique of Protection," "Tariff-Making," "International Cartels," "Combines and Trusts," "Gold Exchange Standard," "Discount Policy," "Immigration Restrictions," etc. It contains *multa*—but unfortunately not *multum*. The program is promising, vast, and imposing; but the execution is disappointing, superficial, and poor.

It is impossible to mention all the shortcomings. There are innumerable theoretical mistakes and even grave offenses against elementary

logic. To cite a few instances, I open the book at random: On page 195 the author defines "dumping" as

selling of the same commodity to a foreign country at a lower price than at home. This dumping may depend on (*a*) an export premium . . . . ; (*b*) a simple lowering of the export price by a monopolistic organisation; (*c*) on inflation selling (*Valutaverschleuderung*), because, for the reason of a depreciating currency, goods are sold in foreign markets cheaper in comparison with the value of money there (. . . . *indem bei sinkender Währung Waren im Ausland im Verhältnis zum dortigen Geldwert billiger angeboten werden*).[2]

Now, (*c*) is obviously not covered by the definition; for, although there is an undercutting of the foreign prices, there is not necessarily a price discrimination between home consumption and export.

Another random sample:

Uniform world-market prices necessarily have a different meaning for different countries, according to the domestic value of money and the purchasing power of the people. Even if the prices of international goods stand at the same level, the national price level may be different and so influence domestic demand in a different way. If the world-market price of a commodity which is needed in a country P is high, in comparison with the purchasing power of the population, then, for this reason, (*a*) the price of these products will have to be high, too, unless costs are again forced down by low wages; (*b*) demand for other goods, domestic as well as foreign, will relatively recede (*zurücktreten*), for the whole amount of the purchasing power changes only slowly and may be treated as constant at that moment; (*c*) prices of other domestic goods may be depressed, because supply presses here to be sold; (*d*) but, also, demand for those urgently needed world-market commodities will be reduced, simply because of the high price. (Example: Poland's demand for cotton is perhaps a case in point.) If, however, the price of a commodity is low in proportion to the purchasing power of a country, the contrary happens, which need not be explained here. There arises, then, for the whole country a consumer's rent, discussed by Marshall, i.e., a national differential gain, because the marginal buyer determines the price and the buyer of the richer country stands above him. Thus, the effect of world-market prices on national price levels is different [p. 42].[2]

[1] It must be noted that the German language of the book is so careless that it is extremely difficult to give a satisfactory translation.

[2] I am compelled to put here the original German text; otherwise the reader would be induced to think that the translation is not correct, for nobody could imagine that anything like the above cited passage could be printed. "Die einheitlichen Weltmarktpreise der Waren bedeuten für die einzelnen Länder notwendig etwas *Verschiedenes* je nach dem inneren Werte des Geldes und der Kaufkraft der Bevölkerung. Auch wenn die Preise für die Waren des Weltmarktes sich

A third example:

The argument of the international division of labor has a consequence, which may easily lead to the contrary of the intended effect, cheapest supply. If every country produces certain goods exclusively or nearly so, it attains a monopolistic position. Let us consider the famous England-Portugal case, according to which the latter supplies only wine and the former, cloth; then there is no reason why Portugal should offer its wines at the lowest costs. It would, on the contrary, demand the highest price which it can charge: it would surely be higher than the mere cost of production. Why should it act otherwise? [p. 96].

This shows clearly the superficiality in using the instruments of economic theory which the author displays on almost every page.

It would, however, be a waste of time and space to multiply these examples. It remains only to observe that the whole arrangement of the book, its subdivision in parts and sections, is also quite illogical and unsuitable. It is difficult to discover any principle, either logical or pedagogical, which could have guided the author in arranging this book. The first part carries the title "International Economic Intercourse" ("Der internationale Wirtschaftsverkehr"); the second, "Foreign Economic Policy"; and the third, "The Economic Significance" ("Die volkswirtschaftliche Bedeutung"). One would expect in Part I a complete theory of international trade. But what one finds is hardly that. After a survey of the historical evolution of international eco-

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gleich hoch stellen, so kann trotzdem das nationale Preisniveau verschieden sein; mithin auch die innere Nachfrage verschieden beinflussen. Steht eine Ware des Weltmarktes, die in einem Lande P gebraucht wird, im Verhältnis zur Kaufkraft der Bevölkerung hoch, so wird dadurch (*a*) der Preis dieser Produkte ebenfalls hoch sein müssen, falls nicht durch niedrige Arbeitslöhne die Kosten wieder herabgedrückt werden, (*b*) die Nachfrage nach anderen Waren sowohl des In- wie des Auslandes wird relativ zurücktreten, da die gesamte Kaufkraft sich nur langsam ändert und in dem Augenblick als konstant angesehen werden kann, (*c*) die Preise anderer Produkte des Inlands können gedrückt sein, weil das Angebot hier zu Verkauf drängt, (*d*) aber auch die Nachfrage nach jenen notwendig gebrauchten Weltmarktwaren wird gerade wegen des hohen Preises eingeschränkt werden. (Beispiel: Polens Nachfrage nach Baumwolle ist vielleicht dafür charakteristisch.) Wenn dagegen eine Ware im Verhältnis zur Kaufkraft des Landes niedrig steht, ist das Umgekehrte der Fall, wie hier nicht ausgeführt werden soll. Es entsteht dann für das ganze Land eine Konsumentenrente, von der Marshall spricht, d. h. ein nationaler Differentialgewinn, weil der Grenzkäufer den Preis bestimmt und die Käufer des reicheren Landes über ihm stehen. Die Wirkung der Weltmarktpreise auf die nationalen Preisniveaus ist also verschieden" (p. 42).

nomic relations, in chapter iii (which contains the foregoing quotation from page 42), there is given an analysis of the items which enter the international balance of payments of a country. Chapter iv deals with the mechanism of the balance of payments and the foreign exchanges. The classical price-specie-flow theory is, naturally, rejected; but what is substituted—shifting of exchange and interest rates—is quite insufficient to explain a permanent equilibrium. Now, one would expect this to be followed by an explanation of the forces underlying the monetary mechanism, viz., what is called "the pure theory of international trade," the theory of comparative cost, or an explanation along the line of mutual supply and demand curves *à la* Marshall, or an extension of the general theory of economic equilibrium to the problems of international trade, such as the Italians Pareto and Barone and their followers attempted. But nothing of that sort happens. The discussion of the theory of comparative cost is scattered over the whole book. The foregoing quotation from page 96, concerning the theory of comparative cost, is incorporated in Part II. An explanation of what the principle of comparative advantage means, and a more detailed discussion, is to be found only at the end of the book, on page 287, after everything, for which the principle is indispensable or at least very useful, as e.g., the long chapter on the effect of import duties (pp. 227–66) or the discussion of the argument that the tariff should equalize cost of production (pp. 123–26), has been disposed of.

With the full responsibility of the reviewer, I must say that this book is only a first draft, which could become a scientific treatise after careful reorganization and rewriting. From the theoretical point of view it is very poor. The historical part is much better, but is also far from being satisfactory, because a just appraisal of historical facts, without a sound theoretical basis, is impossible. The work contains, however, many useful references, facts, and ideas, unfortunately without any systematic arrangement, simply raw material from which one could manufacture a book. It is a somewhat belated product of the younger historical school (Schmoller) and ought, at least in this shape, not to have been written, much less printed.

GOTTFRIED HABERLER

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History of French Colonial Policy, 1870–1925. By STEPHEN H. ROBERTS. 2 vols. London: P. S. King & Son, Ltd., 1929. No. 95 of "Studies in Economics and Political Science of the University of London." Pp. xvi+viii+741. 42*s*.

The expansion of the French colonies within the space of less than two generations from a few weak and scattered units to the second colonial empire of the world, from one million to ten million square kilometers in area, and from five to fifty-five millions in population, is the factual background of Professor Roberts' work. In his first chapter, "General Colonial Policy," he has set himself the task of explaining the growth of the French colonial domain in terms of French political traditions, the industrial development of the mother country, and its relations with its European neighbors. The more detailed analysis of policies he has arranged under three heads: general economic policy (principally the regulation of colonial trade), political policy, and native policy. In these chapters the colonies are considered as a whole in their relations to the mother country, and generalizations are framed as far as possible for all of them. With chapter v, "The Central Organization," they form the first part (pages 1–171) of the two-volume work.

Part II, *French Colonial Theory in Practice*, is descriptive rather than analytical. Taking up each of the colonies in turn, the author treats its political history and economic development, and discusses present-day problems of native policy, land-ownership, communications, export trade, public finance, etc. In the "French Empire since 1914" the author reviews the effects of the war on French colonial policy, and in a final chapter, "A Comparative Study," he brings together in a most interesting fashion the characteristic features of English, French, and Dutch colonial administration. The arrangement although involving a considerable amount of repetition facilitates the use of the book for reference.

Professor Roberts has based his conclusions on a thorough study of source material, including official documents and the contemporary writings of colonial officials, political leaders, and students of colonial problems. References in the text to these sources are abundant, and a full bibliography, classified and critical, is appended. Numerous maps and charts illustrate the various stages of colonial expansion. For a scholarly work of this kind the author's style is surprisingly and re-

freshingly vigorous, even picturesque. The liberal use of figurative language helps to make easy reading, although an incautious reader might sometimes mistake analogy for proof. On page 253 the economic position in present-day Algeria is described thus:

> The economic barque of French colonization in Algeria is close to the rocks, hampered both by storm and its own inherent lack of power, and drifting helpless, in good or bad seas, without immediate hope of getting to land.

Lower down on the same page we are told that Algeria after 1914

> was in the position of a gambler moderately endowed with fortune, hazarding his riches by unwise play and evading a reckoning or even a summing-up of the losses until a catastrophe brought this reckoning and the drastically changed conduct which it implied.

Conclusions are given in straight-from-the-shoulder fashion rather than in the guarded language which has become traditional in scholarly works.

> In the last resort, the French remain Imperialistic visionaries, vacillating between theory and practice. They want Romanization, and every instinct of their being works in the direction of assimilation. Yet they prate of *association* and do everything except introduce a genuine indirect rule! They dabble with widespread economic schemes and plan projects for a systematic *mise en valeur* for the whole of their Empire, but keep away from conditions of hard fact and refuse to tackle the more immediate but less spectacular reforms. . . . . French colonial policy is in this sense large-scale national delusion: it remains for the most part immobile and, at basis, subject to the same principles which were used in dealing with Canada in the days of the *Intendants* [p. 677].

The *History* has the distinction of being the only work in English covering the broad field of modern French colonial effort. It is a valuable supplement to the works in French of Leroy-Beaulieu and Girault. In general the emphasis in Professor Roberts' work is on the political and psychological phases of colonial history and policy. The discussions of economic problems, particularly the fundamental problem of native labor, are less satisfactory.

PERCY WELLS BIDWELL

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Railroad Purchasing and the Business Cycle. By JOHN E. PARTINGTON. Washington, D.C.: Brookings Institution, 1929. Pp. xiii+309. $3.00.

Quite clearly the study of business cycles has reached the monograph stage. Most of the current contributions are either studies of some particular cycle or studies of some particular aspect of the economic order through a series of cycles. Mr. Partington's study belongs to the second group. With the thoroughness of an archaeologist, he has most diligently dug out of the trade journals and manufacturers' records news items concerning orders of locomotives, freight cars, passenger cars, and steel rails over a period of more than half a century. On this basis he has built up quarterly estimates for the period from 1870 to 1926. These records grow increasingly adequate as one comes nearer to the present time, but the underlying assumption of the study is that broadly speaking the picture of cyclical fluctuations is accurate although the totals may not be complete or exact. The author gives an extremely careful discussion of the sources and method and then compares railroad purchasing with business conditions, cycle by cycle. The conclusions are modestly put forward as follows:

As to timing, in a majority of business revivals or recessions studied, railroad buying preceded changes in commercial and industrial activity by some months. This tendency was more marked in recessions than in revivals. Furthermore, it was much more marked in the nineteenth than in the twentieth century. Up to 1900, changes in the volume of rail buying tended to precede similar changes in the volume of orders for other types of equipment. For the years since 1900 no such tendency is evident.

As to amplitude, the degree of fluctuation in railway purchasing with changes in business activity shows no pronounced tendency to change, with the possible exception of rail orders. Of the four types of equipment studied, locomotive purchases showed the greatest tendency to fluctuate with changes in business conditions, and rail purchases the least. Freight-car buying fluctuated almost as much as that of locomotives; passenger-car buying, considerably less.

While the railroads have bought most freely when their earnings were largest, changes in the volume of orders for railway equipment have almost invariably preceded similar changes in the gross earnings. It appears probable, therefore, that the relationship between railway purchases and railway earnings is to be viewed as part of the general

body of interrelationships which make up the business cycle rather than as evidence of a direct dependence of the purchasing activity on the state of the revenues.

There is one question which arises on which the author might have assured his readers, that there is no cyclical variation in the reporting of orders. It certainly is possible that the early revival in railroad orders is accentuated by the fact that railroad manufacturers are eager to tell the world that railroad buying is once more becoming active.

TABLE I

		New Orders		
		Locomotives	Freight Cars	Passenger Cars
1929	I	238	37,913	302
	II	283	21,216	537
	III	151	7,061	238
	IV	376	32,118	185
1930	I	222	27,027	361
	II	47	3,416	155
	III	61	2,694	107

In other words, some manufacturers might report orders at this point in the cycle and never at any other.

It is interesting that this study, quite obviously prepared without reference to any particular situation, should prove extraordinarily useful at the present time. It provides a basis for determining the degree to which the railroads answered President Hoover's appeal that they undertake a program of expansion to prevent the development of a severe depression. On the basis of Mr. Partington's study, one expects a decline in railroad purchases to precede any general decline. As a matter of fact, the recent record of new orders is as shown in Table I. It is very clear that the railroads were purchasing heavily in the last quarter of 1929 and the first quarter of 1930. They did their best.

Willard L. Thorp

Amherst College

New Zealand in the Making: A Survey of Economic and Social Development. By J. B. CONDLIFFE. Chicago: University of Chicago Press, 1930. Pp. 524. $5.00.

Personal inclination, as well as the necessities of a small college, prevented Professor Condliffe from becoming a superspecialist; so, this book will be hard to fit into any library classification. Economics, economic history—these it is chiefly; but no political scientist can afford to miss it, nor can anyone interested in problems of education, while I suspect the sociologist and the anthropologist would not go empty away. A reviewer's fingers itch to type out paragraphs of comment, congratulation, and commiseration of the author's brilliant and brave critique of New Zealand's political, educational, and general intellectual life; but this is a review for economists, so one must let Antipodean fingers do the itching—to get round the throat of one who has so candidly and fearlessly criticized heaven.

Professor Condliffe has set out to do two things. In the first place, he has given us a lucid straightforward account of New Zealand's economic and social history, and brought the story through from the whaling and sealing days of 130 years ago right up to the end of the nineteen-twenties. It is admirably done. We see the beginnings of systematic colonization under the influence of the Wakefield school, for it was in New Zealand that the theories of a century ago were given their best trial. We see the problem of land occupation in face of a native population which was far more capable of resisting than was the case in Australia or even in North America; and one of the best things in the whole book is the account of the way in which, through bloodshed and blunder, the Maori was dispossessed of his land and then began to receive a measure of tardy justice which is opening the way for a promising renaissance. We watch the rise of vast sheep ranches, and of a growing wool export to feed the Yorkshire looms; then comes refrigeration, offering an entirely new world of possibilities to the lands south of the Equator. So there emerges a nation of dairy farmers and lamb-fatteners, organizing production with a remarkable degree of efficiency which has all too often been forgotten in our praises of the Danes, and equipping a transportation and marketing organization to meet the needs of consumers who live halfway round the globe. So today New Zealand is largely what geography, climate, and refrigeration have made her—a land of butter, cheese, frozen mutton, and scenery. Her small size and population have prevented her from following the lead of other big new countries toward ambitious indus-

try-building. Australia, like Canada and the United States, did not rest satisfied till factory chimneys were smoking throughout their lands; but despite much tariff discussion, New Zealand is virtually content to remain chiefly a land of small dairy and sheep farmers.

But to the outside world New Zealand has, for at least a generation, been thought of as something quite different, as the land of radical social experiments, of state enterprises, of daring innovations, of socialistic adventures. Volumes by Pember Reeves, Hammond, Clark, Webb, LeRossignol, and Siegfried, and chapters in Bryce, have established that reputation, of a daring democracy using the state to shape society to its heart's desire. Professor Condliffe's second task is to "debunk" that reputation, and right well he does it. He protests against the exaggerated importance and influence attached to the experiments of the eighteen-nineties, points out that there have been no important new steps—except backward—in the present century, and claims that the comfortable life and prosperity enjoyed by his country before and during the war were the result of the refrigerated industries, efficiency on the farm and in the dairy factory, and the rising level of prices. His thesis is that of course New Zealand was settled by rather hand-picked stock, containing many idealists and humanitarians. Until the end of the seventies it was progressing, and not bothering about social problems—thanks to the boom caused by wool exports and by excessive borrowing and immigration. But in the depression of the eighties, caused by lower prices and the collapse of the borrowing boom, land-hungry farmers and underpaid urban workers pooled their sufferings in a Liberal-Labor political revolt. Their grievances were local, the product of hard times; their common enemy seemed to be the big rancher, the land monopolist; and if they wanted arguments for cudgels they found them in John Stuart Mill, Henry George, and the *Fabian Essays*. So they went to battle and conquered; and while the alliance lasted they were able to push through parliament measures for their mutual benefit—land reform, unlocking the broad acres, closer settlement, factory acts, wages-regulations, and the like. Those reforms, especially the former, did clear the way for, and give impetus to, the economic forces which were transforming New Zealand at that very period. Hence, when the farmer got his land, and refrigeration, cooperative societies, and rising prices made him prosperous, the alliance came to an end, and opposition replaced amity. Prosperity killed any state experimental zeal, the small farmer became conservative in his politics, and Labor has never been strong enough to make any impression on legislation by itself. In other words, falling prices produce

radicalism, rising prices kill it; and from 1896 to the collapse of the boom in 1920 the New Zealand farmer was riding on a blessed price curve.

All this is true. To those who know the Antipodes it has long been evident that the mantle of New Zealand had, early in the present century, blown across the Tasman Sea and fallen on Australia. But it is, nevertheless, also true that New Zealand has always had a much wider conception of the economic functions of government than prevails, for instance, in North America. Necessity, if nothing else, made her undertake the state provision of railroads, telegraph, and more recently "hydro," to say nothing of such other monopolistic public utilities as street cars and gas, while she played a little with insurance, state coal mines, and so forth. And if necessity called for these things, her view of the function of a democratic state enabled her to undertake far-reaching regulation of private enterprises and to embark on extensive and expensive social welfare activities. To anyone brought up in the laissez faire belt, and unaccustomed to precise definition of terms, such activities might well seem "socialistic."

On all these and a score of other aspects of New Zealand's life Professor Condliffe writes with rare ease and charm, and with a quiet pride in his country, yet he never shirks the task when hard things must be said. He is not altogether happy about the present: he fears prosperity came so easily that there has been a diminution of vigor and of efficiency, and that the farmer today is trying to find external reasons for his malaise when the real causes lie in his own surrender to speculation during the palmy days; the lack of any interest in science, in imperial or world-affairs, or in real economic analysis is laid in part at the door of a mediocre, overcentralized, and obsolete educational system; and there is much evidence to show that the country is exporting brains as well as butter.

The hyper-critic may protest that too much space is devoted to some topics, as, for instance, the chapter on wages-regulation and the section on recent troubles in Samoa; he may also object to the practice of putting reference numbers on the same level as the text, instead of lifting them and using smaller type. When he also denounces the practice of putting the footnotes all together at the end of the book instead of at the bottom of the relevant page, the present reviewer joins hands with him, for it is a blemish on a splendidly printed book. But would that all economic and social histories were one-half so readable!

University of Minnesota HERBERT HEATON

Quality as a Determinant of Vegetable Prices. By FREDERICK V. WAUGH. New York: Columbia University Press, 1929. Pp. 154. $2.50.

This volume deals with some of the quality factors in the prices of asparagus, outdoor tomatoes, and hothouse cucumbers on the Boston wholesale market during the course of one season, and discusses the statistical importance of the factors selected for study as determined by the usual multiple-correlation procedure.

Although the object of the study was merely "to discover the quality factors (such as size, shape, color, condition, pack, and other physical characteristics) which influence the prices of locally grown vegetables in the Boston wholesale market, and to measure quantitatively the influence of these factors on price," Dr. Waugh devotes part of chapter ii to a discussion of the economic theory on prices. He thinks that market prices are determined by at least two sets of forces: "first the supply and demand forces which determine the general level of prices of the commodity; and second, the quality factors which determine whether the particular sale in question will be made at prices above or below this general level." The economist is charged with having dealt only with the former and with having neglected the latter by assuming uniform quality. He should now re-examine his assumption of uniform quality "in the same way as he re-examines his assumption of perfect competition and considers the effect of monopoly control." The theoretical economist is likely to reply that "economic theory" is sufficient to cover prices of all grades of goods provided the necessary supply and demand elements are properly represented.

Three brief chapters describe the results obtained for the three perishable commodities. Variations in prices of different lots of asparagus from the current price of a standard grade were found to be related to (1) the amount of green, (2) the average size of the stalks, and (3) uniformity of stalks within the bunch. In the tomato-price variation, "condition, growth, cracks and pack" appeared important, condition being most important. Furthermore, the demand for smaller sizes appeared to be greater on Saturday than on other days. In the case of cucumbers, variations in prices depended on length and diameter.

The results of all such price-analyses need to be appraised in terms of the methods used. Using the usual linear multiple correlation technique, Dr. Waugh is able to point out that 41 per cent of the

variations in asparagus prices for the one season studied were due to green color, 14 per cent to the number of stalks per bunch, 2 per cent to variations in size, or a total of 57 per cent, leaving 43 per cent of the price variations to be accounted for by other quality factors. In the case of tomatoes, as many as thirteen different factors were taken into account and only 48 per cent of the price variations accounted for, leaving 52 per cent to be accounted for by other factors. In the cucumber study, where only two factors were used and the linear quality-price relations converted into curves to improve the correlation, only 58 per cent of the price variations were accounted for while 42 per cent were not.

These percentages are given here as of possible interest to those who have had experience in price analyses, and to those who may be interested in developing further this line of fruitful research. They will probably raise in the minds of the former certain questions of which the latter group should take note. One important question is: How accurately has the nature and importance of any one factor been defined in a linear correlation analysis which accounts for only about half of the price variations? It is an uncomfortable experience of many an analyst that the nature of the influence and the importance of one factor change with the inclusion or exclusion of some of the other factors. This is particularly true in studies where a substantial part of the price fluctuations are still to be accounted for. Unfortunately, there are not available measures of the reliability of the relations between any one factor and the price variations under examination. Studies of the sort contained in this book can at best give only a rough approximation to the nature and extent of the influence of any one factor; and unless the correlation is much above 0.75 one cannot even be certain that the right factors have been "discovered."

Anyone at all acquainted with the very wide range of prices that prevail on any one day for the same commodity, even in the same produce market, will welcome such studies as the one started by Dr. Waugh, not only because they will throw more light on apparent price discrepancies but because they may eventually give growers and dealers more definite knowledge as to consumers' preferences and what they are willing to pay for them.

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The Economic History of England. I. *The Middle Ages.* By E. LIPSON. New York: Macmillan Co., 1929. Pp. viii+552. $5.00.

The appearance of the fifth edition of this work within fourteen years of its publication is at once a tribute to its merits and a proof of enduring interest in medievalism. Apart from an illustration and a few small corrections, however, the fifth edition is identical with the first. The bibliography does not include any material published later than 1914, and little has been changed in the text to meet criticisms directed against the original work. Thus the very sentences on the policy of Edward III remain unaltered in spite of the exception taken to them by the late George Unwin in the *English Historical Review*, XXXI. Yet there can be no doubt that Edward III pursued no consistent policy but merely adopted a series of shifts to meet financial and diplomatic emergencies. This view has recently been demonstrated beyond doubt by the very detailed survey of Anglo-Flemish relations in *The Low Countries and the Hundred Years' War, 1326–1347* by Henry Stephen Lucas. Nevertheless, in spite of the need for rewriting a few pages here and there, the book is still the best on its subject.

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BOOKS RECEIVED

Blakey, Roy G. *Report on Taxation in West Virginia.* (Charleston: Executive Office of the State of West Virginia, 1930.)

Bossard, James H. S., and Dewhurst, J. Frederic. *University Education for Business.* (Philadelphia: University of Pennsylvania Press, 1931.)

Crook, Wilfrid H. *The General Strike.* (Chapel Hill: University of North Carolina Press, 1931.)

Curtis, Roy Emerson. *The Trusts and Economic Control.* (New York: McGraw-Hill Book Co., Inc., 1931.)

Douglas, Edwin R., and Barker, Harry. *A Remedy for Unemployment and Industrial Depression.* (New York: Barker and Wheeler, Engineers, 1931.)

Dubois, Florence. *A Guide to Statistics of Social Welfare in New York City.* (New York: Welfare Council, 1930.)

Dubois-Richard, P. *L'Organisation technique de l'état.* (Paris: Recueil Sirey, 1930.)

Egner, Erich. *Der Sinn des Monopols.* (Berlin: Paul Parey, 1931.)

Elderton, Marion (ed.). *Case Studies of Unemployment.* (Philadelphia: University of Pennsylvania Press, 1931.)

Ely, Richard T. *Hard Times.* (New York: Macmillan Co., 1931.)

Eyskens, Gaston. *Le port de New-York dans son rôle economique.* (Louvain: Rene Fonteyn, 1929.)

Fay, C. R. *Youth and Power.* (London: Longmans, Green & Co., 1931.)

Graham, Frank D. *Exchange, Prices, and Production in Hyper-Inflation: Germany, 1920–1923.* (Princeton: Princeton University Press, 1930.)

Häussermann, Dr. Erich. *Wirtschaftsgemässe Gestaltung der Arbeitslosenversicherung.* (Nürnberg: Krische & Co., 1931.)

Helbing, Albert Theodore. *The Departments of the American Federation of Labor.* (Baltimore: Johns Hopkins Press, 1931.)

Herzfelder, Edmund. *Kreditkontrolle.* (Berlin: Junker und Dünnhaupt, 1930.)

Hoover, Calvin B. *The Economic Life of Soviet Russia.* (New York: Macmillan Co., 1931.)

Jamison, Charles L. *Management of Unit Banks.* Michigan Business Studies, Vol. III, No. 2. (Ann Arbor, 1931.)

League of Nations. *Second Interim Report of the Gold Delegation of the Financial Committee.* (Geneva, 1931.)

Love, Robert A. *Federal Financing.* (New York: Columbia University Press, 1931.)

Marquand, H. A. *The Dynamics of Industrial Combination.* (New York: Longmans, Green & Co., 1931.)

Marschak, Jakob. *Elastizität der Nachfrage.* (Tübingen: J. C. B. Mohr, 1931.)

Megaro, Gaudence. *Vittorio Alfieri, Forerunner of Italian Nationalism.* (New York: Columbia University Press, 1930.)

National Industrial Conference Board, Inc. *Rationalization of German Industry.* (New York, 1931.)

Norton, L. J. *Prices of Illinois Farm Products from 1921 to 1929.* (Urbana: University of Illinois Agricultural Experiment Station, 1930.)

Ohno, Junichi. *Sozialökonomische Theorie des Geldes.* (Leipzig: Werner Scholl, 1931.)

Papi, Giuseppe-Ugo. *L'Influence de l'emission et de l'amortissement des emprunts d'état.* (Bruxelles: Goemaere, 1930.)

Patterson, S. Howard, and Scholz, Karl W. H. *Economic Problems of Modern Life.* (2d ed.) (New York: McGraw-Hill Book Co., Inc., 1931.)

Spahr, Walter Earl. *The Federal Reserve System and the Control of Credit.* (New York: Macmillan Co., 1931.)

Spero, Sterling D., and Harris, Abram L. *The Black Worker.* (New York: Columbia University Press, 1931.)

Sunley, William T., and Pinkerton, Paul W. *Corporation Accounting.* (New York: Ronald Press Co., 1931.)

Theobald, A. D. *Financial Aspects of Subdivision Development.* (Chicago: Institute for Economic Research, 1930.)

Weulersse, G. *Les physiocrates.* (Paris: Gaston Doin & Co., 1931.)

Whittlesey, Charles R. *Governmental Control of Crude Rubber.* (Princeton: Princeton University Press, 1931.)

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